Real-Time Frequency Tracking Using Novel Adaptive Harmonic IIR Notch Filter

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Abstract
In this paper, we propose a simple and effective algorithm for frequency estimation and tracking under a harmonic frequency environment. The proposed filter structure contains only one adaptive coefficient and is very efficient for tracking the fundamental frequency of the periodical signal.

Index terms
Adaptive *IIR Notch filter, harmonics, frequency estimation*

I. Introduction

Frequency tracking of sinusoidal signals using an adaptive IIR notch filter has been a successful approach in many electrical engineering applications [1-4]. However, during a frequency tracking process, we may encounter a situation where a sinusoidal signal may subject to a nonlinear effect in which harmonic frequency components could possibly exist. For example, a signal acquired from a sensor for frequency tracking may contain its harmonic distortion through imperfect amplification at the signal conditioning stage. We may also track frequencies of acoustic signals radiated from various military ground vehicles such as tanks and trucks, as well as trains from which rich harmonics often exhibit. This is due to the transmission of the engine driving force via many mechanical components. Using a standard second-order adaptive IIR notch filter to estimate fundamental and harmonic frequencies is not sufficient since it only tackles one frequency component [1-5]. On the other hand, applying a higher-order adaptive IIR notch filter could result in less efficiency due to the use of multiple adaptive filter coefficients. In this paper, we will develop a simple adaptive harmonic IIR notch filter with a single adaptive coefficient to efficiently perform signal frequency estimation and tracking under a harmonic frequency environment.

II. Adaptive Algorithm Development

We consider an adaptive harmonic IIR notch filter with a single source input as shown below:

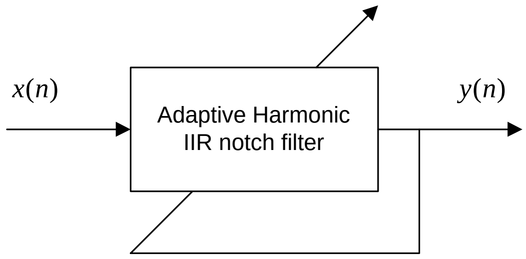

Figure 1 Configuration of the adaptive harmonic IIR notch filter.

As shown in Figure 1, the input $x(n)$ consists of the sinusoidal fundamental frequency component and its harmonics up to the M^{th} order, that is,

$$x(n) = \sum_{m=1}^{M} A_m \sin[2\pi(mf)nT + \varphi_m] + v(n) \quad (1)$$

where A_m, mf, and φ_m are the magnitude, frequency (Hz), and phase angle of the m th harmonic component, respectively. $v(n)$ is a white Gaussian noise. Notice that n and T designate the time index and sampling period. To track the frequency in this harmonic frequency environment, we develop a harmonic IIR notch filter as illustrated in Figure 2 for the case of $M = 3$ (the fundamental component and two harmonics).

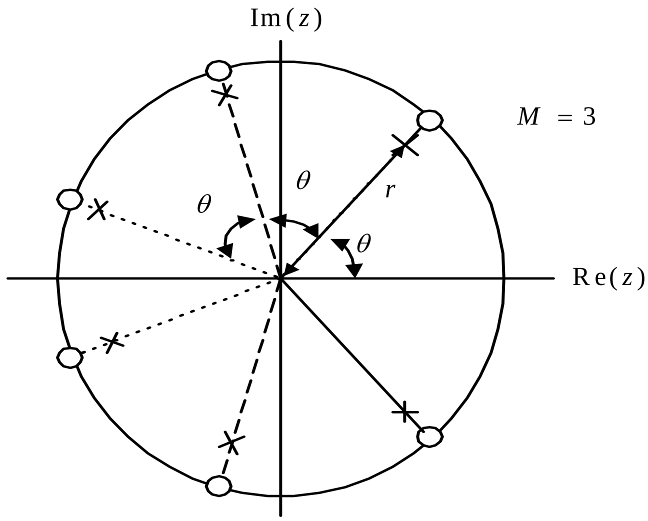

Figure 2. Pole-zero plot for the harmonic IIR notch filter for $M = 3$.

The idea is to place two constrained pole-zero pairs [6] with their angles equal to $\pm m\theta$ (multiple of the fundamental frequency angle θ) relative to the horizontal axis on the pole-zero plot for $m = 1, 2, \cdots, M$, respectively, to construct a notch filter transfer function. The parameter r controls the bandwidth of the notch filter and is chosen to be close to 1 to achieve a narrowband notch. Hence, once θ is adapted to the angle corresponding to the fundamental frequency, each $m\theta$ ($m > 1$) will automatically lock its harmonic frequency. Therefore, we can construct the filter transfer function in a cascaded form as

$$H(z) = \frac{Y(z)}{X(z)} = \prod_{m=1}^{M} H_m(z) \tag{2}$$

where the transfer function $H_m(z)$ at the mth filter section is defined as

$$H_m(z) = \frac{1 - 2z^{-1}\cos(m\theta) + z^{-2}}{1 - 2rz^{-1}\cos(m\theta) + r^2 z^{-2}} \tag{3}$$

From (2) and (3), and knowing that the transfer function has only one adaptive coefficient θ, we can easily determine the filter output $y_m(n)$ at the mth filter section as

$$\begin{aligned} &\text{For } m = 1, 2, \cdots, M \\ &y_m(n) = y_{m-1}(n) - 2\cos(m\theta) y_{m-1}(n-1) + y_{m-1}(n-2) \\ &\qquad + 2r\cos(m\theta) y_m(n-1) - r^2 y_m(n-2) \end{aligned} \tag{4}$$

Again, notice that $y_0(n) = x(n)$ and $y(n) = y_M(n)$. The power of the filter output $y_M(n)$ from the last filter section is to be minimized via the adaptive algorithm.

Next, we simply apply the LMS algorithm to obtain the update equation of the filter coefficient. Taking the derivative of the instantaneous power of the filter output at the last section, $y_M^2(n)$, and setting the result to zero, we achieve

$$\theta(n+1) = \theta(n) - 2\mu y_M(n) \beta_M(n) \tag{5}$$

where μ is a convergence parameter and the gradient term $\beta_m(n)$ at section m is defined as

$$\beta_m(n) = \frac{\partial y_m(n)}{\partial \theta(n)} \tag{6}$$

with

$$\beta_0(n) = \frac{\partial y_0(n)}{\partial \theta(n)} = \frac{\partial x(n)}{\partial \theta(n)} = 0 \tag{7}$$

Using (4), $\beta_m(n)$ can be recursively computed using the following equation:

$$\begin{aligned} &\text{For } m = 1, 2, \cdots, M \\ &\beta_m(n) = \beta_{m-1}(n) - 2\cos[m\theta(n)]\beta_{m-1}(n-1) \\ &\qquad + 2m\sin[m\theta(n)] y_{m-1}(n-1) + \beta_{m-1}(n-2) \\ &\qquad + 2r\cos[m\theta(n)]\beta_m(n-1) - r^2 \beta_m(n-2) \\ &\qquad - 2rm\sin[m\theta(n)] y_m(n-1) \end{aligned} \tag{8}$$

Finally, we could convert $\theta(n)$ in radians to the desired estimated fundamental frequency in Hz as follows:

$$f(n) = \frac{\theta(n)}{2\pi} \times f_s \text{ (Hz)} \tag{9}$$

IV. Experiments

In order to illustrate the developed algorithm, we presented the following simulation example. The input signal $x(n)$ of 1000 Hz plus two harmonics, sampled at $f_s = 8000$ Hz, is given as

$$\begin{aligned} x(n) = {} & \sin[2\pi \times 1000 \times n / f_s] \\ & + 0.5\cos[2\pi \times (2\times 1000) \times n / f_s] \\ & - 0.25\cos(2\pi \times (3\times 1000) \times n / f_s) + v(n) \end{aligned} \tag{10}$$

The signal to noise power ratio is set to 18 dB. We used $M = 3$ and $r = 0.95$ for constructing the harmonic notch filter, and chose $\theta(0) = 0.225\pi$ radians (corresponding to 900 Hz) and $\mu = 0.0001$ for the LMS algorithm. Figure 3 shows the time-domain input and output at each filter section of the harmonic notch filter and we observed that the algorithm converges after 150 LMS iterations.

Figure 4(a) shows the obtained frequency magnitude response, which has null points located at the fundamental and harmonic frequencies. Figure 4 (b) illustrates frequency tracking of the desired fundamental frequency $f(n)$ as defined in (9). In this example, we can identify the fundamental frequency as 1000 Hz.

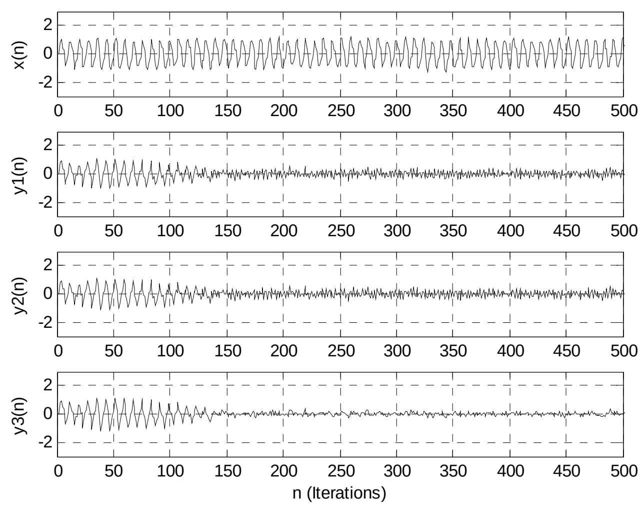

Figure 3. Input and outputs for the M = 3 IIR notch filter sections.

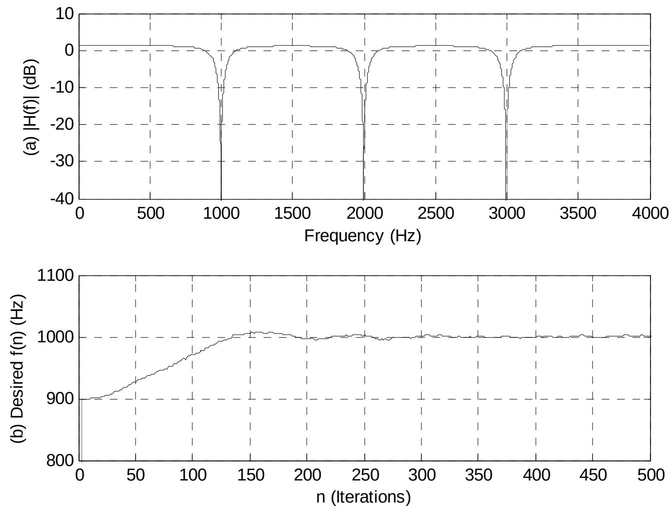

Figure 4. Frequency magnitude response and frequency tracking.

Our other simulations also demonstrated that the harmonic notch filter possesses a frequency tracking capability when the input signal has fewer harmonics than we anticipate. On the other hand, if the number of harmonics in the input signal is greater than that of what we expect, the harmonic notch filter can still track the fundamental signal frequency but with some performance degradation.

Next, we conducted a real time implementation of the adaptive filter with the setup shown in Figure 5. We set the sampling rate to 8000 Hz for the TI TMS320C6711 DSP board and used a floating-point format for algorithm implementation. The signal to be tracked is assumed to have a fundamental frequency along with second and third harmonic components produced from the distortion channel. In this experiment, we obtained the distorted signal by filtering a sawtooth signal with a fundamental frequency around 1000 Hz using a low-pass filter with a cut-off frequency of 1500 Hz. During the tracking process, the fundamental frequency was initially set to 1000 Hz and it varied between 900 Hz and 1300 Hz. Our adaptive harmonic IIR notch filter employed $M = 3$, $r = 0.95$, $\theta(0) = 0.225\pi$ radian (selection of the initial coefficient was based on prior knowledge about the frequency to be tracked), and $\mu = 1\times 10^{-14}$. The oscilloscope display shown in Figure 5 depicts the distorted sinusoid with second and third harmonics (top signal in the display) and the converged adaptive filter output $y_M(n)$ from the last filter section (bottom signal in the display). At the same time, we monitored the desired estimated frequency using our adaptive harmonic IIR notch filter, in which our developed algorithm tracked the desired frequency at 1096.997 Hz.

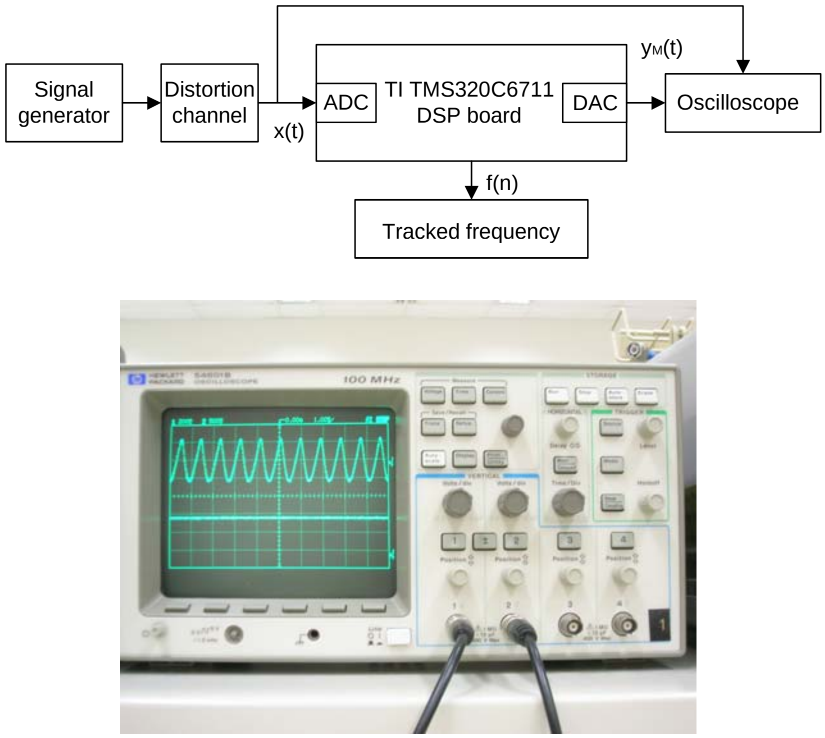

Figure 5. Experiment for implementing the adaptive harmonic IIR notch filter.

Figure 6 lists a partial code segment of our implementation of the adaptive harmonic IIR notch filter. The real time application demonstrates the efficiency of our developed algorithm and practical applications of using the adaptive harmonic IIR notch filter are currently under investigation.

```
volatile int sample;
float r=0.95; /* parameter to control the notch filter bandwidth */
float x[3]={0.0,0.0,0.0};
float y1[3]={0.0,0.0,0.0};
float y2[3]={0.0,0.0,0.0};
float y3[3]={0.0,0.0,0.0};
float bt0[3]={0.0,0.0,0.0};
float bt1[3]={0.0,0.0,0.0};
float bt2[3]={0.0,0.0,0.0};
float bt3[3]={0.0,0.0,0.0};
float theta=0.225*3.14;
float mu=0.00000000000001; /* convergence parameter */
float f;

interrupt void AtoD()
{
   int i;
   sample=mcbsp0_read();  /* ADC, sampling rate = 8 kHz*/
   //adaptive harmonic IIR notch filter
   for(i=2; i>0; i--)   /* update input buffer */
   {   x[i]=x[i-1];   }
   x[0]=(float) sample; /* load input sample */
   for (i=2;i>0;i--)    /* update output buffer for each section IIR filter */
   {    y1[i]=y1[i-1];
         y2[i]=y2[i-1];
        y3[i]=y3[i-1]; }
   for (i=2;i>0;i--)  /* update gradient buffer for each section */
   {    bt0[i]=bt0[i-1];
```

```
        bt1[i]=bt1[i-1];
        bt2[i]=bt2[i-1];
       bt3[i]=bt3[i-1]; }
/* perform harmonic IIR notch filter */
   y1[0]=x[0]-2*cos(theta)*x[1]+x[2]+2*r*cos(theta)*y1[1]-r*r*y1[2];
   y2[0]=y1[0]-2*cos(2*theta)*y1[1]+y1[2]+2*r*cos(2*theta)*y2[1]-r*r*y2[2];
   y3[0]=y2[0]-2*cos(3*theta)*y2[1]+y2[2]+2*r*cos(3*theta)*y3[1]-r*r*y3[2];
/* perform gradient update */
   bt1[0]=bt0[0]-
2*cos(1*theta)*bt0[1]+2*sin(1*theta)*x[1]+bt0[2]+2*r*cos(theta)*bt1[1]-
r*r*bt1[2]-2*r*sin(theta)*y1[1];
   bt2[0]=bt1[0]-
2*cos(2*theta)*bt1[1]+2*2*sin(2*theta)*y1[1]+bt1[2]+2*r*cos(2*theta)*bt2[1]-
r*r*bt2[2]-2*r*2*sin(2*theta)*y2[1];
   bt3[0]=bt2[0]-
2*cos(3*theta)*bt2[1]+2*3*sin(3*theta)*y2[1]+bt2[2]+2*r*cos(3*theta)*bt3[1]-
r*r*bt3[2]-2*r*3*sin(3*theta)*y3[1];
   /* apply LMS algorithm*/
   theta=theta-2*mu*y3[0]*bt3[0];
   f=theta*8000/(2*3.1415926); /* convert theta to frequency in Hz */
   sample= y3[0]; /* send the output from the last notch filter to DAC */
}
```

Figure 6. Code segment sample for the adaptive harmonic IIR notch filter.

V. Conclusions

In this paper, we have developed a novel adaptive harmonic IIR notch filter for frequency estimation and tracking, which only uses a single adaptive coefficient and can effectively track the sinusoidal signal under a harmonic frequency environment. We expect that the filter developed could be useful in many applications.

Acknowledgements

The authors would like to thank anonymous reviewers for their valuable comments and contributions.

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