Calculating the Capacitance and Inductance of Multiconductor Transmission Lines

by

Sarhan M. Musa and Matthew N.O. Sadiku
College of Engineering
Prairie View A&M University
Prairie View, Texas 77446, USA

Abstract: *Multiconductor transmission lines in multilayered circuit are used to reduce dimensions and minimize pulse distortion and crosstalk for high-speed and high-density digital applications. The accurate and efficient results of the self and coupling capacitance and inductance can help the designers to optimize the layout of the integrated circuits. In this paper we present modeling of the capacitance and inductance of four multiconductor transmission lines: two conductors in two different dielectric layers, two conductors in same dielectric layers, three conductors in three different dielectric layers, and three- line bus in a layered dielectric. The modeling and simulation is implemented using finite element method.*

Keywords- Multiconductor transmission lines, capacitance, inductance, finite element method, modeling

Introduction

Designing microstrip components that operate at high levels of radio frequency (RF) power and designing of overlay microwave couplers often depend on multilayer microstrip. Therefore, analysis and electrical characterization of multiconductor transmission lines have been a major focus for researchers in integrated circuit (IC) and very large scale integration (VLSI) technologies. It is no doubt, that advances in IC technology have caused renewed interest and attraction in the computation of capacitance and inductance of multiconductor transmission lines.

Previous attempts at the problem include using the method of moments (MoM) [1,2], spectral domain approach (SDA) [3,4], Green's function approach [3,6], the method of lines (MoL) [5,7], domain decomposition method (DDM), and finite difference methods (FDM) [7] . In this work, we use finite element method with COMSOL multiphysics package to calculate the capacitance per unit length and inductance per unit length of open multiconductor transmission lines. We use finite-element method (FEM) in modeling the transmission lines structure, because FEM is especially suitable for the computation of electric and electromagnetic fields in strongly inhomogeneous media.

Also, it has high computation accuracy and fast computation speed. We consider four microstrip systems: (1) two conductors in two different dielectric layers, (2) two conductors in same dielectric layers, (3) three conductors in three different dielectric layers, and (4) three-line bus in a layered dielectric. We compare our results with previous investigators and find them to be close.

Discussion and Results

In any electromagnetic field analysis the placement of far-field boundary is an important concern, especially when dealing with open solution regions. It is necessary to take into account that the natural boundary of a line at infinity and the presence of remote objects and their potential influence on the field shape [8]. In all our simulations, the open microstrip line is surrounded by a w X h shield, where w is the width and h is the thickness.

The inductance and capacitance per unit length of muticonductor transmission lines are related as

$$\left[L\right] = \mu_o \varepsilon_o \left[C_o\right]^{-1} \qquad (1)$$

Where

$\left[L\right]$ = inductance matrix.

$\varepsilon_o =$ permittivity of free space or vacuum.

$\mu_o =$ permeability of free space or vacuum.

$\left[C_o\right]^{-1}$ = the inverse matrix of the capacitance of the multiconductor transmission line when all dielectric constants are set equal to 1.

The modeling and simulation of the lines were taken for the following four cases of two conductors in two different dielectric layers, two conductors in same dielectric layers, three conductors in three different dielectric layers, and three-line bus in a layered dielectric.

A. Two conductors in two different dielectric layers

Figure 1 shows the cross section for two conductors in two different dielectric layers with the following parameters:

ε_1 = dielectric constant = 1

ε_2 = dielectric constant = 6.8

w_1 = width of the strip line = 0.2mm

w_2 = width of the strip line = 0.2mm

h_1 = height of conductor 1 from the ground = 0.6mm

h_2 = height of conductor 2 from the ground = 0.2mm

t_1 = thickness of conductor 1 = 0.1mm

t_2 = thickness of conductor 2 = 0.2mm

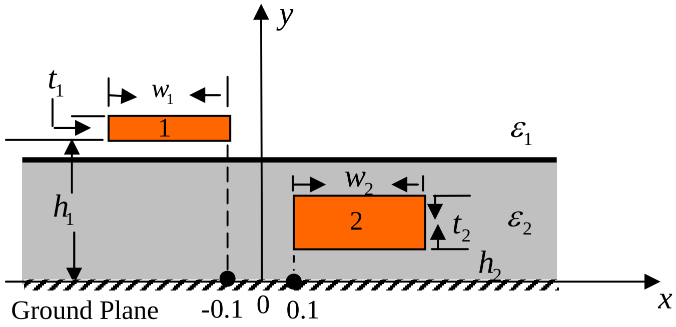


Figure 1. Cross-section of two conductors in two different dielectric layers.

The geometry is enclosed by a 3 X 7 mm shield. The simulation was done twice – once to calculate $[C]$ when $\varepsilon_2 = 6.8$ and the other to calculate $[C_o]$ when $\varepsilon_2 = 1.0$. We need $[C_o]$ to calculate the inductance matrix $[L]$ according to eq. (1). The mesh consists of 2684 elements, as shown in Fig. 2. Figure 3 shows potential distribution in contour plot, while Fig. 4 shows the potential distribution for y = 1mm.

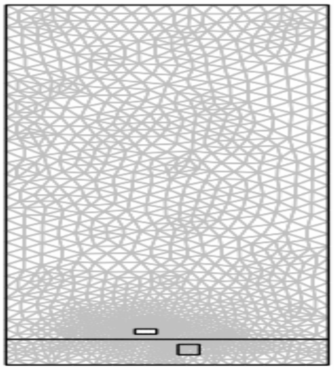


Figure 2. Mesh of two conductors in two different dielectric layers using node 1 as input.

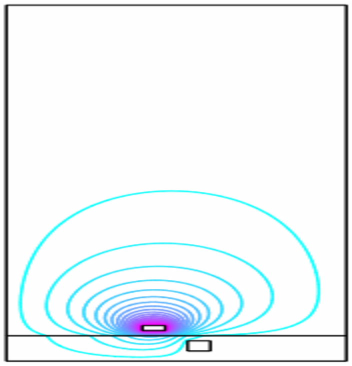


Figure 3. Contour plot of two conductors in two different dielectric layers using node 1 as input.

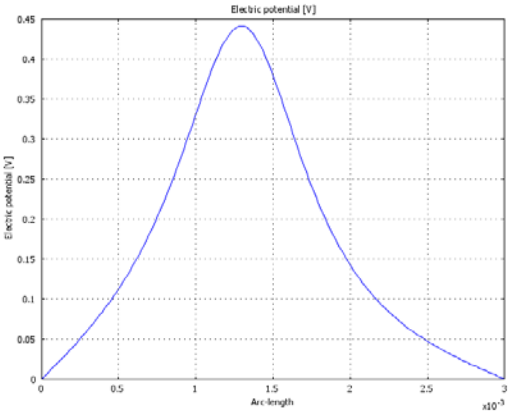


Figure 4. Potential distribution of two conductors in two different dielectric layers using port 1 as input at y = 1mm.

Table 1 shows the finite element results for the capacitance per unit length and the inductance per unit length of two conductors in two different dielectric layers. The results in Table 1 are compared with the work of previous investigations. They are in good agreement.

Table 1: Values of the capacitance (in F/m) and inductance (in H/m) coefficients for two conductors in two different dielectric layers.

C and L	Reference [1]	Reference [2]	Reference [3]	Reference [4]	Reference[5]	Our Work
C_{11}	$0.3651\text{x}10^{-10.}$	$0.3701\text{x}10^{-10}$	$0.3640\text{x}10^{-10}$	$0.3779\text{x}10^{-10}$	$0.3521\text{x}10^{-10}$	$0.3853\text{x}10^{-10}$
C_{12}	$-0.1562\text{x}10^{-10}$	$-0.1520\text{x}10^{-10}$	$-0.1580\text{x}10^{-10}$	$0.1571\text{x}10^{-10}$	$-0.1550\text{x}10^{-10}$	$-0.1572\ \text{x}10^{-10}$
C_{22}	$0.2099\text{x}10^{-9}$	$0.2108\text{x}10^{-9}$	$0.2078\text{x}10^{-9}$	$0.2151\text{x}10^{-9}$	$0.2064\text{x}10^{-9}$	$0.2185\ \text{x}10^{-9}$
L_{11}	$0.5315\text{x}10^{-6}$	$0.5403\text{x}10^{-6}$	-	-	-	$0.4935\ \text{x}10^{-6}$
L_{12}	$0.1241\text{x}10^{-6}$	$0.1229\text{x}10^{-6}$	-	-	-	$0.1110\ \text{x}10^{-6}$
L_{22}	$0.3235\text{x}10^{-6}$	$0.3204\text{x}10^{-6}$	-	-	-	$0.3068\ \text{x}10^{-6}$

B. Two conductors in the same dielectric layers

Figure 5 shows the cross section for two conductors in the same dielectric layers with the following parameters:

ε_1 = dielectric constant = 1

ε_2 = dielectric constant = 6.8

w_1 = width of the strip line = 0.2mm

w_2 = width of the strip line = 0.2mm

h_1 = height of conductor 1 from the ground = 0.6mm

h_2 = height of conductor 2 from the ground = 1.0mm

t_1 = thickness of conductor 1 = 0.1mm

t_2 = thickness of conductor 2 = 0.1mm

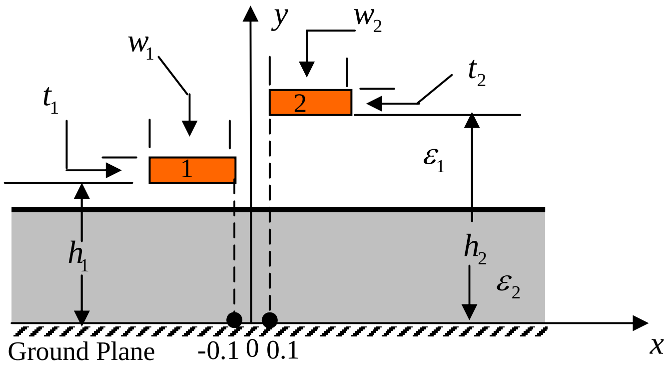


Figure 5. Cross-section of two conductors in same dielectric layers.

The geometry is enclosed by a 3 X 10 mm shield. The mesh consists of 2752 elements, as shown in Fig. 6. Figure 7 shows a 2-D surface potential distribution using node 1 as input, while Fig. 8 shows the potential distribution for y = 1mm.

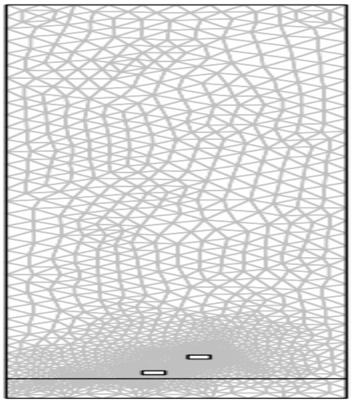


Figure 6. Mesh of two conductors in same dielectric layers.

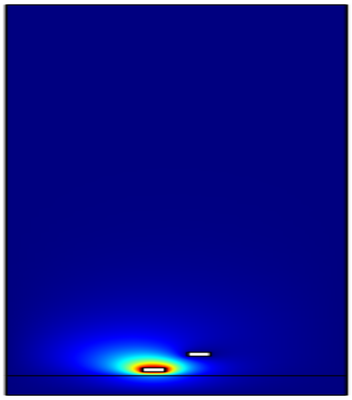


Figure 7. 2-D surface potential distribution of two conductors in same dielectric layers using node 1 as input.

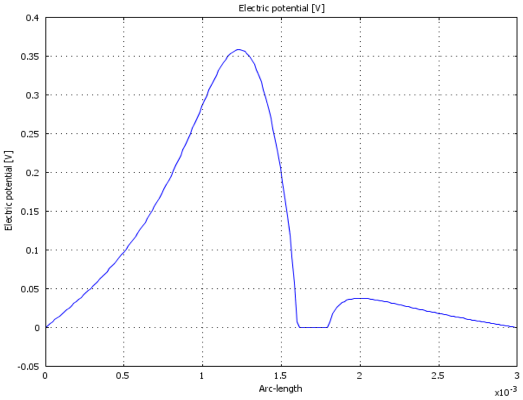


Figure 8. Potential distribution of two conductors in same dielectric layers using port 1 as input at y = 1mm.

Table 2 shows the finite element results for the capacitance per unit length and inductance per unit length of two conductors in same dielectric layers. The results in Table 2 are compared with the work of previous investigations. They are in good agreement.

Table 2: Values of the capacitance (in F/m) and inductance (in H/m) coefficients for two conductors in same dielectric layers

C and L	Reference [1]	Reference [2]	Our Work
C_{11}	$0.3720\text{x}10^{-10}$	$0.3757\text{x}10^{-10}$	$0.3889\ \text{x}10^{-10}$
C_{12}	$-0.6889\text{x}10^{-11}$	$-0.6657\text{x}10^{-11}$	$-0.6638\text{x}10^{-11}$
C_{22}	$0.2169\text{x}10^{-10}$	$0.2217\text{x}10^{-10}$	$0.2361\ \text{x}10^{-10}$
L_{11}	$0.5437\text{x}10^{-6}$	$0.5501\text{x}10^{-6}$	$0.5051\ \text{x}10^{-6}$
L_{12}	$0.2244\text{x}10^{-6}$	$0.2235\text{x}10^{-6}$	$0.1828\text{x}10^{-6}$
L_{22}	$0.6368\text{x}10^{-6}$	$0.6407\text{x}10^{-6}$	$0.5622\ \text{x}10^{-6}$

C. Three conductors in three different dielectric layers

Figure 9 shows a more complex geometry of the cross section for three conductors in three different dielectric layers with the following parameters:

ε_1 = dielectric constant = 4.5
ε_2 = dielectric constant = 1
ε_3 = dielectric constant = 6.8
w_1 = width of the strip line = 0.2mm
w_2 = width of the strip line = 0.2mm
h_1 = height of conductor 1 from the ground = 0.6mm
h_2 = height of conductor 2 from the ground = 1mm
t_1 = thickness of conductor 1 = 0.1mm
t_2 = thickness of conductor 2 = 0.1mm
d = diameter of conductor 3 = 0.3 mm

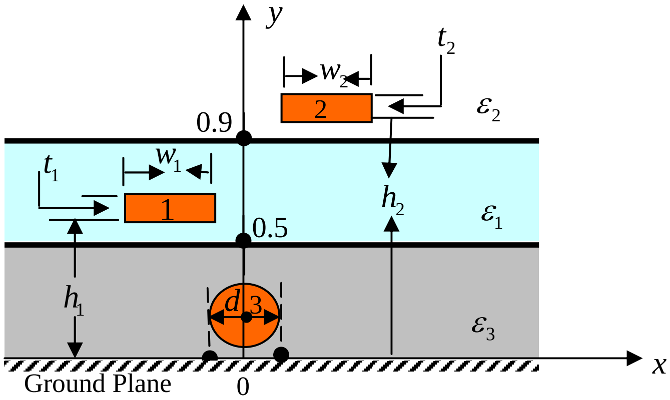


Figure 9. Cross-section of three conductors in three different dielectric layers.

The geometry is enclosed by a 3 X 10 mm shield. The simulation was done twice – once to calculate $[C]$ when $\varepsilon_1 = 4.5$, $\varepsilon_3 = 6.8$ and the other to calculate $[C_o]$ when $\varepsilon_1 = \varepsilon_3 = 1.0$. We need $[C_o]$ to calculate the inductance matrix $[L]$ according to eq. (1). The mesh consists of 4580 elements, as shown in Fig. 10. Figure 11 shows the potential distribution in streamline plot for node 3 (circular conductor), while Fig. 12 shows the potential distribution for y = 1mm.

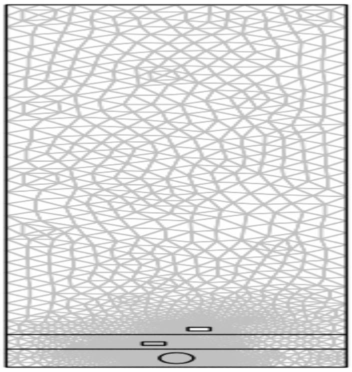


Figure 10. Mesh of three conductors in three different dielectric layers.

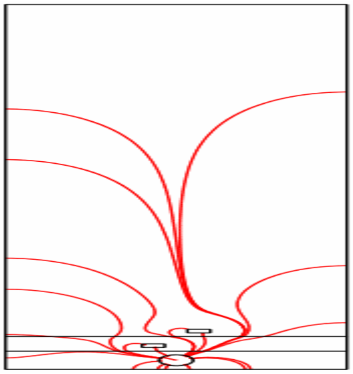


Figure 11. Potential distribution in streamline plot for node 3 (circular conductor).

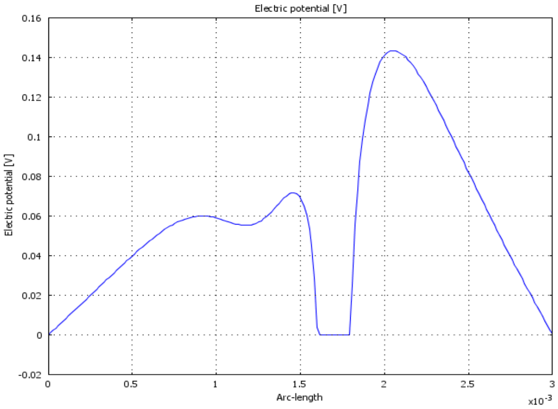


Figure 12. Potential distribution of three conductors in three different dielectric layers at y =1mm.

Table 3 shows the finite element results for the capacitance per unit length and inductance per unit length of three conductors in three different dielectric layers. The results in Table 3 are compared with the work of previous investigations. They are in good agreement.

Table 3: Values of the capacitance (in F/m) and inductance (in H/m) coefficients for three conductors in three different dielectric layers

C and L	Reference [1]	Reference [4]	Reference [6]	Reference [7]	Our Work
C_{11}	0.1244x10^{-9}	0.1250 x10^{-9}	0.1259x10^{-9}	0.1308x10^{-9}	0.1283x10^{-9}
C_{12}	-0.1300x10^{-10}	-0.1296x10^{-10}	-0.1313x10^{-10}	-	-0.1271 x10^{-10}
C_{22}	0.3340x10^{-10}	0.3388x10^{-10}	0.3410x10^{-10}	0.3524x10^{-10}	0.3486 x10^{-10}
C_{13}	-0.6825x10^{-10}	-0.6927x10^{-10}	-0.6956x10^{-10}	-	-0.7184 x10^{-10}
C_{23}	-0.7196x10^{-11}	-0.7204x10^{-11}	-0.7182x10^{-11}	-	-7.1326 x10^{-11}
C_{33}	0.3523 x10^{-9}	0.3572x10^{-9}	0.3576x10^{-9}	0.3759x10^{-9}	0.7740x10^{-9}
L_{11}	0.4965x10^{-6}	0.4948x10^{-6}	0.4919x10^{-6}	0.4711x10^{-6}	0.4622 x10^{-6}
L_{12}	0.1996x10^{-6}	0.1992x10^{-6}	0.1989x10^{-6}	-	0.1609 x 10^{-6}
L_{22}	0.6163x10^{-6}	0.6155x10^{-6}	0.6128x10^{-6}	0.5824x10^{-6}	0.5493 x 10^{-6}
L_{13}	0.1183x10^{-6}	0.1178x10^{-6}	0.1775 x10^{-6}	-	0.1061 x 10^{-6}
L_{23}	0.07728x10^{-6}	0.0770x10^{-6}	0.0768x10^{-6}	-	0.0616 x 10^{-6}
L_{33}	0.2331x10^{-6}	0.2302x10^{-6}	0.2299x10^{-6}	0.2174x10^{-6}	0.2149 x 10^{-6}

D. Three- line bus in a layered dielectric

Figure 13 shows the cross section for three-line bus in a layered dielectric with the following parameters:

ε_1 = dielectric constant = 4.3

ε_2 = dielectric constant = 3.2

ε_3 = dielectric constant = 1

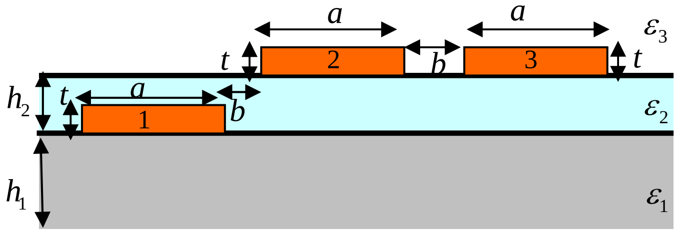
a = width of each the strip line = 350 μm

b = distance between any two strip lines = 150 μm

h_1 = height of conductor 1 from the ground = 200 μm

h_2 = height of conductors 2 and 3 from the first substrate = 100 μm

t = thickness of the conductors = 70 μm



Ground Plane

Fig. 13. Cross section of three-line bus in a layered dielectric.

The geometry is enclosed by a 3500 X 2000 μm shield. The simulation was done twice – once to calculate $[C]$ when ε_1 and ε_2 equal to 4.3 and 3.2 respectively and the other to calculate $[C_o]$ when ε_1 and ε_2 are set equal to 1. We need $[C_o]$ to calculate the inductance matrix $[L]$ according to eq. (1). Figure 14 shows contour plot of three-line bus in a layered dielectric using node 2 as input.

The capacitance matrix of the multiconductor transmission line $[C_o]$ when all dielectric constants are set equal to 1 is

$$C_o = \begin{bmatrix} 44.26 & -11.11 & -0.5606 \\ -11.11 & 41.98 & -12.66 \\ -0.5606 & -12.66 & 38.23 \end{bmatrix} \text{(pF/m)} \qquad (2)$$

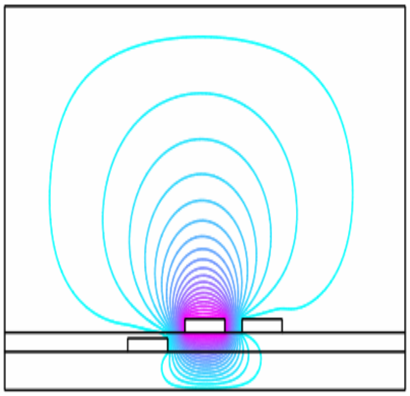


Fig. 14. Contour plot of three-line bus in a layered dielectric using node 2 as input.

Table 4 shows the finite element results for the capacitance per unit length and inductance per unit length of three-line bus in a layered dielectric. The results in Table 4 are compared with the work of a previous investigation [6]. They are in good agreement.

Table 4: Values of the capacitance (in pF/m) and inductance (in nH/m) coefficients for three-line bus in a layered dielectric

C and L	Reference [6]	Our Work
C_{11}	142.09	142.62
C_{12}	-21.765	-21.67
C_{22}	93.529	94.43
C_{13}	-0.8920	-0.6455
C_{23}	-18.098	-18.61
C_{33}	87.962	88.35
L_{11}	277.7	271.8
L_{12}	87.8	81.3
L_{22}	328.6	318.3
L_{13}	36.8	326.4
L_{23}	115.8	106.6
L_{33}	338.0	326.4

Conclusion

In this paper, we have presented the modeling of four microstrip systems: two conductors in two different dielectric layers, two conductors in same dielectric layers, three conductors in three different dielectric layers, and three-line bus in a layered dielectric. The results obtained using COMSOL for the capacitance per unit length and inductance per unit length agree well with those found in the literature.

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