Using Concrete Proportioning to Teach Spreadsheet Modeling

by

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Abstract: *A detailed linear programming formulation was derived following the American Concrete Institute (ACI) 211.1-91 Standard Practice for Selecting Proportions for Normal Heavyweight, and Mass Concrete. Linearization is included for all constraints including those given in tabular form. Constraints were included to account for slump, strength, replacement of cement with pozzolan, and air entrainment admixture. The air entrainment admixture requirement assumes a linear form for air entrainment versus admixture dosage. Implementation in Excel using the Solver add-in yielded results consistent with examples given in ACI 211.1-91. A procedure for implementing the formulation using Excel Solver is provided. This teaching example demonstrates the complementary combination of regression analysis and linear programming.*

I. Introduction

Concrete is a major component of many, if not most, construction projects. Optimal concrete proportioning involves selecting the optimum amounts of constituents (i.e., cement, water, fine aggregate, coarse aggregate, pozzolans, and admixtures) to obtain concrete that meets strength and workability requirements at the lowest cost. Linear Programming (LP) refers to solving constrained optimization problems having an objective function and constraint equations that are linear functions of decision variables.

Applications of linear programming in the concrete industry have included optimization of the cement blend (Xirokostas & Zoppas [1]), optimization of the aggregate selection (Lewis [2]), and concrete proportioning (Kasperkiewicz [3]). The objective of each of these works was to minimize cost while achieving required material properties.

To apply LP to the problem of concrete proportioning, therefore requires developing linear functions of decision variables that represent the cost and constraints as shown by

Kasperkiewicz. However, this paper differs from the work of Kasperkiewicz in the following ways:

- Pozzolan (fly ash) is included as a decision variable,
- Air entrainment admixture is included as a decision variable,
- Coarse aggregate composition is assumed to be given.
- The spreadsheet applied to examples is Microsoft Excel, rather than QuattroPro, and
- Reference data is from American references, rather than Polish sources.

This paper presents the linearization and application of constraint equations provided in ACI 211.1-91. Where relationships are given in the form of tabular data, regression was used to develop an equivalent equation comparable to those developed by Jerath & Kabbani [4]. Implementation of the LP formulation in Excel is given for the two examples in ACI 211.1-91 [5], that is non-air entrained concrete, and air entrained concrete. Although the examples do not directly incorporate pozzolan, both examples are given with and without fly ash in the mix.

II. Linear Programming Formulation

The linear programming (LP) formulation includes the objective function and constraint equations defined in the following sections. The standard linear programming constraint has a linear function of decision variables on the left-hand side and a constant on the right hand side of an equation or inequality [6].

Objective Function

The objective is to make concrete that satisfies strength and other properties at a minimum cost. The objective function for cost for one cubic yard (meter) of concrete, *Z*, is given in Equation 1.

$$Z = c_W W_{add} + c_C C_{mix} + c_P P + c_{CA} C_{A,dry} + c_{FA} F_{A,dry} + c_{AE} A_E \qquad (1)$$

The decision variables and unit cost parameters are defined as follows:

W_{add} = Pounds (kilogram) of water added to mix per cubic yard (meter) of concrete
C_{mix} = Pounds (kilogram) of cement per cubic yard (meter) of concrete in the final mix
P = Pounds (kilogram) of pozzolan per cubic yard (meter) of concrete
$C_{A,dry}$ = Pounds (kilogram) of dry course aggregate per cubic yard (meter) of concrete
$F_{A,dry}$ = Pounds (kilogram) of dry fine aggregate per cubic yard (meter) of concrete

A_E = Ounces (milliliter) of air entrainment admix per cubic yard (meter) of concrete
c_W = Cost per pound (kilogram) of water
c_C = Cost per pound (kilogram) of cement
c_P = Cost per pounds (kilogram) of fly ash
$c_{CA,dry}$ = Cost per pound (kilogram) of dry coarse aggregate
$c_{FA,dry}$ = Cost per pound (kilogram) of dry fine aggregate
c_{AE} = Cost per ounce air entrainment admixture

Pozzolan and entrained air admixture terms can be omitted from Equation 1 for a basic mix.

Slump Constraint

Step 1 of ACI 211.1-91 is to choose the slump, S, which results in the constraint of Equation 2.

$$S = S_{req} \tag{2}$$

For a given aggregate size, linear regression can be used to model the relationship between slump and weight of mix water, W_{mix}, based on data in ACI 211.1-91 Tables 6.3.3 and A1.5.3.3 as in Equation 3, where s_S is the slope, and i_S is the intercept. Substituting Equation 3 into 2 and rearranged to obtain the LP constraints results in Equation 4.

$$S = s_S W_{mix} + i_S \tag{3}$$

$$s_S W_{mix} = S_{req} - i_S \tag{4}$$

Strength Constraint

Abrams' Law for compression strength (f_c) as a function of the water to cement weight ratio (W/C) is given in Equation (5) where A and B are parameters assumed constant for a particular type of cement.

$$f_c = \frac{A}{B^{W/C}} \tag{5}$$

Equation 5 can be expressed as a linear function of the decision variables W and C as in Equation 6. The inequality was introduced to accommodate higher than the required strength, which is the case when the water to cement ratio is constrained for severe exposure environments.

$$\left[\ln\left(f_c\right) - \ln\left(A\right)\right]C + \ln\left(B\right)W_{mix} \leq 0 \tag{6}$$

The subscript *mix* was added to *W* to indicate the variable accounts for all water in the mix regardless of the source (e.g., water added and aggregate moisture). The parameters $\ln\left(B\right)$ and $\ln\left(A\right)$ in this equation can be found using regression on data in ACI 211.1-91 Tables 6.3.4(a) and A1.5.3.4(a) (f_c as a function of W/C) using Equation 7 as the regression model.

$$\ln\left(f_c\right) = \ln\left(A\right) - \ln\left(B\right)\left(\frac{W}{C}\right) \tag{7}$$

Water to Cement Ratio Constraint

ACI 211.1-91 Tables 6.3.4(b) and A1.5.3.4(b) give the maximum permissible water-cement ratio or water-cementitious ratio for concrete in severe exposures. This specification results in the constraint given by Equation 8.

$$\frac{W_{mix}}{C_{mix} + P} \leq \left[\frac{W_{mix}}{C}\right]_{\max} \tag{8}$$

Where $\left[\frac{W_{mix}}{C}\right]_{\max}$ is the maximum water to cementitious ratio by weight. Equation 8 can be rearranged into a linear form as shown in Equation 9.

$$W_{mix} - \left[\frac{W_{mix}}{C}\right]_{\max} C_{mix} - \left[\frac{W_{mix}}{C}\right]_{\max} P \leq 0 \tag{9}$$

The pozzolan term can be omitted from Equation 9 for a basic mix.

Pozzolanic Material Constraints

The fraction of pozzolan, P, to total cementitious materials, $C_{mix} + P$, by weight, F_w in Equation 10, can be written as another LP constraint as in Equation 11.

$$F_w = \frac{P}{C_{mix} + P} \tag{10}$$

$$F_w C_{mix} + (F_w - 1)P = 0 \tag{11}$$

It may be necessary to replace cement at a ratio exceeding 1 to 1 to obtain strengths equivalent to straight cement mix [7]. Equation 12 represents this additional constraint, where K_{pc} is the weight replacement ratio and C_{rep} is the cement replaced from the equivalent straight cement mix. Equation 12 is rewritten in LP form as Equation 13.

$$P = K_{pc} C_{rep} \tag{12}$$

$$P - K_{pc} C_{rep} = 0 \tag{13}$$

Equation 14 ties the replaced cement, C_{rep}, and remaining cement, C_{mix}, to the straight cement weight, C.

$$C = C_{mix} + C_{rep} \tag{14}$$

Equation 14 is rearranged to the LP form as Equation 15.

$$C - C_{mix} - C_{rep} = 0 \tag{15}$$

Coarse Aggregate Volume Constraint

The volume of oven-dry-rodded coarse aggregate per unit volume of concrete is given in ACI 211.91.1-91 Tables 6.3.6 and A1.5.3.6. The corresponding constraint can be written as Equation 16 where $V_{CA,dry}$ is the volume of dry coarse aggregate and V_{total} is the total concrete volume. Substituting for the dry coarse aggregate volume in terms of the decision variable dry coarse aggregate weight and rearranging in LP form results in Equation 17 where $\rho_{CA,dry}$ is the dry coarse aggregate density.

$$\frac{V_{CA,dry}}{V_{total}} = \left[\frac{V_{CA,dry}}{V_{total}}\right]_{table} \tag{16}$$

$$\frac{1}{\rho_{CA,dry}} C_{CA,dry} = \left[\frac{V_{CA,dry}}{V_{total}}\right]_{table} V_{total} \tag{17}$$

Wet and Dry Aggregate Constraints

Equations 18 and 19 relate the dry aggregate weights ($C_{A,dry}$, $F_{A,dry}$) to the wet aggregate weights ($C_{A,wet}$, $F_{A,wet}$) for coarse and fine aggregate, respectively. Although these equations can be used to eliminate the dry or wet aggregate variable from other equations, they are included in the LP formulation to simplify other equations. The parameters w_{CA} and w_{FA} represent the total moisture content of coarse and fine aggregate, respectively.

$$C_{A,wet} - C_{A,dry}\left(1 + w_{CA}\right) = 0 \tag{18}$$

$$F_{A,wet} - F_{A,dry}\left(1 + w_{FA}\right) = 0 \tag{19}$$

Entrained Air Constraint

The recommended entrained air content for various environments (mild, moderate, or severe exposure to freezing or deicing agents) are given in ACI 211.91.1-91 Tables 6.3.3 and A1.5.3.3. The air entrainment constraint is written as Equation 20, where $A_{entrained}$ is the fraction of the total volume of concrete composed of entrained air.

$$A_{entrained} = [A_{entrained}]_{req} \tag{20}$$

The air entrainment is assumed to be a linear function of the volume of admix used per 100 pounds (kilograms) of cementitious material, a_e, as shown in Equation 21, where s_{ea} is the slope and i_{ea} is the intercept.

$$A_{entrained} = s_{ea} a_e + i_{ea} \tag{21}$$

The ounces (milliliters) of admix per 100 pounds (kilograms) of cement, a_e, can be written in terms of the decision variable, A_E, as shown in Equation 22. Equations 20, 21, and 22 can be combined to form Equation 23. Rearranging yields the LP constraint form in Equation 24.

$$a_e = \frac{A_E}{[(C_{mix} + P)/100]} \tag{22}$$

$$s_{ea} \frac{A_E}{[(C_{mix} + P)/100]} + i_{ea} = [A_{entrained}]_{req} \tag{23}$$

$$s_{ea} A_E + \left[\frac{i_{ea} - [A_{entrained}]_{req}}{100}\right] C_{mix} + \left[\frac{i_{ea} - [A_{entrained}]_{req}}{100}\right] P = 0 \tag{24}$$

The pozzolan term can be omitted from Equation 24 for a basic mix.

Volume Constraint

Equation 25 constrains the total volume of water, cement, pozzolan, coarse aggregate, fine aggregate, and air entrainment admixture to the concrete volume, V_{total}, minus entrapped and entrained air.

$$\begin{aligned} &\frac{1}{\rho_W} W_{add} + \frac{1}{\gamma_C \rho_W} C_{mix} + \frac{1}{\gamma_P \rho_W} P + \frac{1}{\gamma_{CA,dry} \rho_W} C_{A,dry} + \frac{1}{\gamma_{FA,dry} \rho_W} F_{A,dry} + K_{AE} A_E \\ &= V_{total} (1 - A_{entrapped} - A_{entrained}) \end{aligned} \tag{25}$$

where material properties are defined as follows:

ρ_W = Density of water
γ_C = Specific gravity of cement
$\gamma_{CA,dry}$ = Specific gravity of dry coarse aggregate
$\gamma_{FA,dry}$ = Specific gravity of dry fine aggregate
γ_P = Specific gravity of pozzolan

Pozzolan and entrained air admixture terms can be omitted from Equation 25 for a basic mix. The conversion factor K_{AE} is included to convert from ounces (milliliter) of air entrainment admixture, A_E, to cubic ft (meter) of concrete. The entrapped air, $A_{entrapped}$, and entrained air, $A_{entrained}$, are given in ACI 211.91.1-91 Tables 6.3.3 and A1.5.3.3.

Mixing Water Constraint

Aggregate moisture not absorbed by the aggregate is available as mixing water. Equation 26 summarizes the total mixing water available, where w_{CA} is the total moisture content of coarse aggregate, a_{CA} is the coarse aggregate absorption, w_{FA} is the total moisture content of fine aggregate, and a_{FA} is the fine aggregate absorption. Rearranged into LP form, Equation 26 becomes Equation 27.

$$W_{mix} = W_{add} + (w_{FA} - a_{FA})F_{A,dry} + (w_{CA} - a_{CA})C_{A,dry} \tag{26}$$

$$W_{mix} - W_{add} - (w_{FA} - a_{FA})F_{A,dry} - (w_{CA} - a_{CA})C_{A,dry} = 0 \tag{27}$$

Non-Negativity Constraints

The non-negativity constraints are imposed for all decision variables. Excel Solver has an option to enforce the non-negativity constraint, which is explained in the Appendix, so the equation forms are not included here.

III. Results and Discussion

The Linear Programming model presented above was applied to ACI 211.1-91 Example 1 and Example 2 with and without pozzolan. Equations applicable to these examples are listed in Table 1.

Table 1. Linear Programming Equation Applicability

ACI 211.1-91 Concrete Proportioning Example	**Mix**	**Linear Programming Equation Set**
Example 1: Non-air entrained concrete in benign conditions	Straight Cement	1, 4, 6, 17, 18, 19, 25, and 27
	Cement/ Pozzolan	1, 4, 6, 11, 13, 14, 17, 18, 19, 25, and 27
Example 2: Air entrained concrete exposed to severe conditions	Straight Cement	1, 4, 6, 9, 17, 18, 19, 24, 25, and 27
	Cement/ Pozzolan	1, 4, 6, 9, 11, 13, 14, 17, 18, 19, 24, 25, and 27

Since Example 2 uses all of the LP equations formulated above, only the Example 2 spreadsheets are included in this paper. However, Example 1 setup and results are included below. The following unit costs were assumed for both examples:

- Cost per pound (kilogram) of water (c_W): \$0.000157/lb (\$0.0003462/kg)
- Cost per pound (kilogram) of cement (c_C): \$0.050/lb (\$0.1102/kg)
- Cost per pounds (kilogram) of pozzolan (fly ash) (c_P): \$0.015/lb (\$0.03307/kg)
- Cost per pound (kilogram) of dry coarse aggregate ($c_{CA,dry}$): \$0.0055/lb (\$0.01213/kg)
- Cost per pound (kilogram) of dry fine aggregate ($c_{FA,dry}$): \$0.006/lb (\$0.01323/kg)
- Cost per ounce (milliliter) of air entrainment admixture (c_{AE}): \$0.015625/oz (\$0.0005283/ml)

Material properties common to both examples are listed below:

- Density of water (ρ_W): 62.4 lbs/ft^3 (1000 kg/m^3)
- Specific gravity of cement (γ_C): 3.15
- Specific gravity of dry coarse aggregate ($\gamma_{CA,dry}$): 2.68
- Density of dry coarse aggregate ($\rho_{CA,dry}$): 100 lbs/ft^3 (1600 kg/m^3)
- Specific gravity of dry fine aggregate ($\rho_{CA,dry}$): 2.64
- Specific gravity of pozzolan (fly ash) (γ_P): 2.25

The procedure for using Excel Solver, which was used to solve the linear programming problem, is described in the Appendix.

ACI 211.1-91 Example 1

ACI 211.1-91 Example 1 develops proportions for non-air entrained concrete not exposed to severe weathering or sulfate attack. Equation 3 slump regression parameters are i_S = -20.50 and s_S = 0.08 for slump in inches and i_S = -520.7 and s_S = 3.4252 for slump in millimeters based on data from ACI 211.1.-91 Table 6.3.3 for non-air entrained concrete with 1.5 inch (38 mm) nominal maximum size coarse aggregate and slump ranging from 1 to 4 inches (25 to 102 mm). The regression error is 0.5 inches (12.5 mm) of slump because the same water data applies to two values of slump 1 inch (25 mm) apart (Table 2).

Table 2 - Slump Regression Results for Nominal Maximum Coarse Aggregate Size of 1.5 inches (38 mm) in Non-Air Entrained Concrete

Slump, in (mm)	Water, lb/yd^3 (kg/m^3)	Predicted Slump, in (mm)
1 (25)	275 (163)	1.5 (38)
2 (51)	275 (163)	1.5 (38)
3 (76)	300 (178)	3.5 (89)
4 (102)	300 (178)	3.5 (89)

The desired slump for this example is 3 to 4 inches, so a required slump, S_{req}, of 3.5 inches (89 mm) was used for the LP formulation. Equation 7 strength model regression parameters for non-air entrained concrete based on ACI 211.1-91 Table 6.3.4(a) are $\ln(A)$ = 9.8011 and $-\ln(B)$ = -2.6647 for strength in psi and $\ln(A)$ = 4.8242 and $-\ln(B)$ = -2.6647 for strength in MPa. These parameters result in compression strength errors of less than 2% relative to the ACI data (Table 3).

Table 3 – Non-Air Entrained Concrete Strength Regression Results

W/C by Weight (Mass)	Compressive Strength at 28 Days, psi (MPa)	
	ACI Data	Prediction
0.41	6000 (41)	6055 (42)
0.48	5000 (34)	5024 (35)
0.57	4000 (28)	3953 (27)
0.68	3000 (21)	2949 (20)
0.82	2000 (14)	2031 (14)

For a fine aggregate fineness modulus of 2.8 and 1.5 inch (38 mm) nominal maximum size coarse aggregate, ACI 211.1-91 Table 6.3.6 gives $\left[\frac{V_{CA,dry}}{V_{total}}\right]_{table}$ = 0.71 ft^3 (m^3) of coarse aggregate per ft^3 (m^3) of concrete. Material properties specific to this example are as follows:

- Total coarse aggregate moisture content (w_{CA}): 2.0%

- Total fine aggregate moisture content (w_{FA}): 6.0%
- Coarse aggregate absorption (a_{CA}): 0.5%
- Fine aggregate absorption (a_{FA}): 0.7%
- Entrapped air ($A_{entrapped}$): 1%

Tables 4 and 5 show the comparison of LP results to those given in ACI 211.1-91 Example 1. The LP implementation resulted in the constraints being satisfied with the strength equal to the limiting value of 3500 psi (24.13 MPa). Minor differences between results in U.S. Customary Units and results in SI Units are due to numerical rounding of some parameters in the LP implementations. The mixes are nearly identical to ACI results. The pozzolan results, using a fraction of pozzolan by weight, F_w, equal to 0.20 and a weight replacement, K_{pc}, of 1.2, show a redistribution of the cement to mix cement and pozzolan, resulting in a reduction in cost.

Differences between the LP and the ACI results are due to the slightly different use of data from the strength table. In the ACI example, the strength of 3500 psi (24.13 MPa) was used to interpolate for the water to cement ratio between 3000 and 4000 psi (20.69 and 27.58 MPa). In the current example, regression was used to model all of the strength data, resulting in a strength of 3461 psi (23.87 MPa) for the ACI mix based on the regression model, which is 1% off from the target strength. Using the two data points that bound the strength requirement for the regression model (effectively interpolating) would eliminate this difference.

Table 4 – ACI 211.1-91 Example 1 Quantity Summary

Method	**Component Quantity, lbs/CY (kg/m^3)**									
	W mix	**W add**	**C**	**C mix**	**C rep**	**P**	**CA dry**	**CA wet**	**FA dry**	**FA wet**
LP Straight Cement	300 (178)	202 (120)	-	487 (289)	-	-	1917 (1137)	1955 (1160)	1315 (780)	1394 (827)
LP Cement/Pozzolan	300 (178)	204 (121)	487 (289)	403 (239)	84 (50)	101 (60)	1917 (1137)	1955 (1160)	1267 (751)	1343 (796)
ACI 211.1-16 Based on Weight	300 (178)	199 (118)	-	484 (287)	-	-	1917 (1137)	1955 (1160)	1369 (812)	1451 (861)
ACI 211.1-16 Based on Volume	300 (178)	199 (118)	-	484 (287)	-	-	1917 (1137)	1955 (1160)	1318 (782)	1451 (861)

Table 5 – ACI 211.1-91 Example 1 Property Summary

Method	Cost, $/CY ($/m^3)	W/C	Strength, psi (MPa)	VCAdry/V	Slump, in (mm)	Volume, ft^3 (m^3)
LP **Straight Cement**	42.83 (55.92)	0.62	3500 (24.13)	0.71	3.5 (89)	27.00 (1.00)
LP **Cement/Pozzolan**	39.85 (52.12)	0.60	3697 (25.49)	0.71	3.5 (89)	27.00 (1.00)
ACI 211.1-16 **Based on Weight**	42.99 (56.22)	0.62	3461 (23.87)	0.71	3.5 (89)	27.31 (1.01)
ACI 211.1-16 **Based on Volume**	42.68 (55.82)	0.62	3461 (23.87)	0.71	3.5 (89)	27.00 (1.00)

ACI 211.1-91 Example 2

ACI 211.1-91 Example 2 develops proportions for air entrained concrete subject to severe weathering or sulfate attack, resulting in a water to cement ratio limit of 0.5. Equation 3 slump regression parameters are i_S = -20.10 and s_S = 0.08 for slump in inches and i_S = -510.54 and s_S = 3.4252 for slump in millimeters based on data from ACI 211.1.-91 Table 6.3.3 for air entrained concrete with 1 inch (25.4 mm) nominal maximum size coarse aggregate and slump ranging from 1 to 4 inches (25 to 102 mm). The regression error is 0.5 inches (12.5 mm) of slump because the same water data applies to two values of slump 1 inch (25 mm) apart (Table 6).

Table 6 - Slump Regression Results for Nominal Maximum Coarse Aggregate Size of 1 inches (25.4 mm) in Air Entrained Concrete

Slump, in (mm)	Water, lb/yd^3 (kg/m^3)	Predicted Slump, in (mm)
1 (25)	270 (160)	1.5 (38)
2 (51)	270 (160)	1.5 (38)
3 (76)	295 (175)	3.5 (89)
4 (102)	295 (175)	3.5 (89)

The desired slump for this example is 1 to 2 inches, so a required slump, S_{req}, of 1.5 inches (38 mm) was used for the LP formulation. Equation 7 strength model regression parameters for air entrained concrete based on ACI 211.1-91 Table 6.3.4(a) are $\ln(A)$ = 9.5886 and $-\ln(B)$ = -2.6859 for strength in psi and $\ln(A)$ = 4.6116 and $-\ln(B)$ = -2.6859 for strength in MPa.

These parameters result in compression strength errors of less than 0.5% relative to the ACI data (Table 7). For a fine aggregate fineness modulus of 2.8 and 1 inch (25.4

mm) nominal maximum size coarse aggregate, ACI 211.1-91 Table 6.3.6 gives

$$\left[\frac{V_{CA,dry}}{V_{total}}\right]_{table} = 0.69\ \text{ft}^3\ (\text{m}^3)\text{ of coarse aggregate per ft}^3\ (\text{m}^3)\text{ of concrete.}$$

Table 7 – Air Entrained Concrete Strength Regression Results

W/C by Weight (Mass)	Compressive Strength at 28 Days, psi (MPa)	
	ACI Data	**Prediction**
0.40	5000 (34.48)	4985 (34.37)
0.48	4000 (27.58)	4021 (27.73)
0.59	3000 (20.69)	2993 (20.63)
0.74	2000 (13.79)	2000 (13.79)

Sample air entrainment model parameters (Equation 21) of i_{ea} = 0 and s_{ea} = 0.04 air entrainment per ounce of admixture per 100 pounds of cement (0.00067 air entrainment per milliliters of admixture per 100 kilograms of cement) were used based on an air entrainment admixture dosage, a_e, of 1.5 ounces (90 milliliters) producing 6% entrained air ($A_{entrained}$) [8]. Material properties specific to this example are as follows:

- Total coarse aggregate moisture content (w_{CA}): 3.0%
- Total fine aggregate moisture content (w_{FA}): 5.0%
- Coarse aggregate absorption (a_{CA}): 0.5%
- Fine aggregate absorption (a_{FA}): 0.7%
- Entrained air ($A_{entrained}$): 6%

The spreadsheet in U.S. Customary Units (SI Units) representing the LP formulation is shown in Figure 1 (Figure 2). Numbers in the first column labeled “Eq.” indicate the equation numbers corresponding to the spreadsheet rows. The constraints are satisfied with the strength equal to the limiting value of 3000 psi (20.69 MPa).

This example was also analyzed assuming some cement would be replaced with fly ash to reduce costs. The fraction of pozzolan by weight, F_w, was set to 0.20 and the weight replacement value, K_{pc}, was set to 1.2. The spreadsheet in U.S. Customary Units (SI Units) representing the LP formulation is shown in Figure 3 (Figure 4).

Fig. 1 – ACI 211.1-91 Example 2 LP Spreadsheet

(Straight Cement Mix, U.S. Customary Units)

	W mix (lbs/ CY)	W add (lbs/CY)	C (lbs/ CY)	CA dry (lbs/ CY)	CA wet (lbs/ CY)	FA dry (lbs/ CY)	FA wet (lbs/ CY)	AE (oz/ CY)
Quantity	270	178	540	1809	1845	1232	1306	8.10

Eq.	Minimize Total Cost	W mix ($/lb)	W add ($/ lb)	C ($/lb)	CA dry ($/lb)	CA wet ($/lb)	FA dry ($/lb)	FA wet ($/lb)	AE ($/oz)	Total Cost ($/CY)
1	Unit Costs		0.000157	0.050000	0.005500		0.006000		0.015625	44.50

Eq.	Subject to	Wmix	Wadd	C	CA dry	CA wet	FA dry	FA wet	AE	LHS		RHS
4	Slump	0.080								21.60	=	21.60
6	Strength	2.686		-1.582						-129.21	<=	0.00
9	Max W/C	1		-0.5						0.00	<=	0
17	Coarse Aggregate				0.010					18.09	=	18.09
18	CA Wet and Dry				-1.020	1.000				0.00	=	0.00
19	FA Wet and Dry						-1.060	1.000		0.00	=	0.00
24	Air Entrainment			-0.0006					0.04	0.00	=	0
25	Volume	0.01603		0.005	0.006		0.006		0.001044	25.38	=	25.38
27	Water	1.000	-1.000		-0.015		-0.053			0.00	=	0.00

Fig. 2 – ACI 211.1-91 Example 2 LP Spreadsheet

(Straight Cement Mix , SI Units)

	W mix (kg/ m^3)	W add (kg/ m^3)	C (kg/ m^3)	CA dry (kg/ m^3)	CA wet (kg/ m^3)	FA dry (kg/ m^3)	FA wet (kg/ m^3)	AE (ml/ m^3)
Quantity	160	105	320	1073	1095	731	775	286.83

Eq.	Minimize Total Cost	W mix ($/kg)	W add ($/kg)	C ($/kg)	CA dry ($/kg)	CA wet ($/kg)	FA dry ($/kg)	FA wet ($/kg)	AE ($/ml)	Total Cost ($/ m^3)
1	Unit Costs		0.000346	0.110200	0.012130		0.013230		0.000528	58.17

Eq.	Subject to	Wmix	Wadd	C	CA dry	CA wet	FA dry	FA wet	AE	LHS		RHS
4	Slump	3.425								548.54	=	548.54
6	Strength	2.686		-1.582						-76.56	<=	0.00
9	Max W/C	1		-0.5						0.00	<=	0
17	Coarse Aggregate				0.001					0.67	=	0.67
18	CA Wet and Dry				-1.020	1.000				0.00	=	0.00
19	FA Wet and Dry						-1.060	1.000		0.00	=	0.00
24	Air Entrainment			-0.0006					0.00067	0.00	=	0
25	Volume	0.00100	0.00100	0.00032	0.00037		0.00038		0.000001	0.94	=	0.94
27	Water	1.000	-1.000		-0.015		-0.053			0.00	=	0.00

Fig. 3 – ACI 211.1-91 Example 2 LP Spreadsheet

(Cement/Pozzolan Mix, U.S. Customary Units)

	W mix (lbs/ CY)	W add (lbs/ CY)	C (lbs/ CY)	C mix (lbs/ CY)	C rep (lbs/ CY)	P (lbs/ CY)	CA dry (lbs/ CY)	CA wet (lbs/ CY)	FA dry (lbs/ CY)	FA wet (lbs/ CY)	AE (oz/ CY)
Quantity (LP)	270.00	164.75	522.00	432.00	90.00	108.00	1809.00	1845.18	1473.89	1562.33	8.100025

Eq.	**Minimize Total Cost**	**W mix ($/lb)**	**W add ($/lb)**	**C ($/lb)**	**C mix ($/lb)**	**C rep ($/lb)**	**P ($/lb)**	**CA dry ($/lb)**	**CA wet ($/lb)**	**FA dry ($/lb)**	**FA wet ($/lb)**	**AE ($/oz)**	**Total Cost ($/ CY)**
1	Unit Costs		0.000157		0.05		0.015000	0.005500		0.006000		0.015625	42.17

Eq.	**Subject to**	**W mix**	**W add**	**C**	**C mix**	**C rep**	**P**	**CA dry**	**CA wet**	**FA dry**	**FA wet**	**AE**	**LHS**		**RHS**
4	Slump	0.080											21.60	=	21.60
6	Strength	2.686		-1.582									-100.73	<=	0.00
9	Max w/c	1			-0.5		-0.5						0.00	<=	0
11	Pozzolan (fly ash)				0.2		-0.8						0.00	=	0
13	Pozzolan (fly ash)					-1.2	1						0.00	=	0
14	Cement			1	-1	-1							0.00	=	0
17	Coarse Aggregate							0.010					18.09	=	18.09
18	CA Wet and Dry							-1.020	1.000				0.00	=	0.00
19	FA Wet and Dry									-1.060	1.000		0.00	=	0.00
24	Air Entrainment				-0.0006		-0.0006					0.04	0.00	=	0
25	Volume		0.016		0.005088		0.007	0.006		0.006		0.001044	25.38	=	25.38
27	Water	1.000	-1.000					-0.015		-0.053			0.00	=	0.00

Fig. 4 – ACI 211.1-91 Example 2 LP Spreadsheet

(Cement/Pozzolan Mix, SI Units)

	W mix (kg/ m^3)	W add (kg/ m^3)	C (kg/ m^3)	C mix (kg/ m^3)	C rep (kg/ m^3)	P (kg/ m^3)	CA dry (kg/ m^3)	CA wet (kg/ m^3)	FA dry (kg/ m^3)	FA wet (kg/ m^3)	AE (ml/ m^3)
Quantity (LP)	160	98	310	256	53	64	1073	1095	875	927	289

Eq.	Minimize Total Cost	W mix ($/kg)	W add ($/kg)	C ($/kg)	C mix ($/kg)	C rep ($/kg)	P ($/kg)	CA dry ($/kg)	CA wet ($/kg)	FA dry ($/kg)	FA wet ($/kg)	AE ($/ml)	Total Cost ($/ m^3)
1	Unit Costs		$0.000346		0.1102		$0.033070	$0.012130		$0.013230		$0.000528	$55.13

Eq.	Subject to	W mix	W add	C	C mix	C rep	P	CA dry	CA wet	FA dry	FA wet	AE	LHS		RHS
4	Slump	3.425											548.54	=	548.54
6	Strength	2.686		-1.582									-59.67	<=	0.00
9	Max w/c	1			-0.5		-0.5						0.00	<=	0
11	Pozzolan (fly ash)				0.2		-0.8						0.00	=	0
13	Pozzolan (fly ash)					-1.2	1						0.00	=	0
14	Cement			1	-1	-1							0.00	=	0
17	Coarse Aggregate							0.001					0.67	=	0.67
18	CA Wet and Dry							-1.020	1.000				0.00	=	0.00
19	FA Wet and Dry									-1.060	1.000		0.00	=	0.00
24	Air Entrainment				-0.0006		-0.0006					0.00067	0.00	=	0
25	Volume		0.001		0.000318		0.00044	0.00037		0.00038		0.000001	0.94	=	0.94
27	Water	1.000	-1.000					-0.015		-0.053			0.00	=	0.00

Tables 8 and 9 show the comparison of LP results to those given in ACI 211.1-91 Example 2. Minor differences between results in U.S. Customary Units and results in SI Units are due to numerical rounding of some parameters in the LP implementations. The mixes are nearly identical to ACI results. The pozzolan results show a redistribution of the cement to mix cement and pozzolan, resulting in a reduction in cost. The water to cement ration limit of 0.5 drove the strength to 3811 psi (26.28 MPa), much higher than the 3000 psi (20.69 MPa) required.

Table 8 – ACI 211.1-91 Example 2 Quantity Summary

Method	**Component Quantity, lbs/CY (kg/m^3)**										**AE, oz/CY (ml/ m^3)**
	W mix	**W add**	**C**	**C mix**	**C rep**	**P**	**CA dry**	**CA wet**	**FA dry**	**FA wet**	
LP Straight Cement	270 (160)	178 (105)	-	540 (320)	-	-	1809 (1073)	1845 (1095)	1232 (731)	1306 (775)	8.1 (286.8)
LP Cement/Pozzolan	270 (160)	165 (98)	522 (310)	432 (256)	90 (53)	108 (64)	1809 (1073)	1845 (1095)	1474 (875)	1562 (927)	8.1 (286.8)
ACI 211.1-16 Based on Weight	270 (160)	170 (101)	-	540 (320)	-	-	1791 (1063)	1771 (1051)	1321 (784)	1387 (823)	-
ACI 211.1-16 Based on Volume	270 (160)	170 (101)	-	540 (320)	-	-	1791 (1063)	1771 (1051)	1321 (784)	1387 (823)	-

Table 9 – ACI 211.1-91 Example 2 Property Summary

Method	Cost, $/CY ($/m^3)	W/C	Strength, psi (MPa)	VCAdry/V	Slump, in (mm)	Volume, ft^3 (m^3)	Entrained Air (%)
LP Straight Cement	44.50 (58.17)	0.50	3811 (26.28)	0.67	1.5 (38)	27.00 (1.00)	6.0
LP Cement/Pozzolan	42.17 (55.13)	0.50	3811 (26.28)	0.67	1.5 (38)	26.99 (1.00)	6.0
ACI 211.1-16 Based on Weight	44.80 (58.59)	0.50	3811 (26.28)	0.66	1.5 (38)	27.42 (1.02)	6.0
ACI 211.1-16 Based on Volume	44.80 (58.59)	0.50	3811 (26.28)	0.66	1.5 (38)	27.42 (1.02)	6.0

IV. Summary and Conclusions

Linear programming was shown to be useful for concrete proportioning using a linearization of ACI 211.1-91 constraints including those given in tabular form. The examples were extended to include pozzolan and air entrainment admixture. Results were consistent with those provided in the ACI examples, which indicates that the examples inherently minimize cost. Linear programming provided a straightforward method for imposing constraints relating to replacement of cement with pozzolan for reducing costs. Once programmed, it is simple to change parameter values and rerun Solver to obtain a new set of optimum mixture proportions. This paper provides a blend of regression analysis, constraint linearization, and linear programming optimization applied to concrete proportioning. Since it is based on a commonly used and understood construction material, this example can readily capture the interest of students learning spreadsheet modeling techniques.

V. Acknowledgements

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Appendix: Spreadsheet Implementation

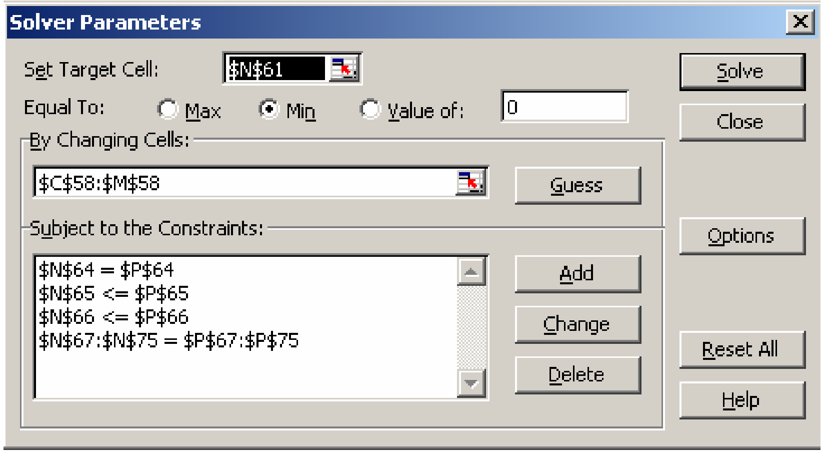
Implementing the linear programming problem in Excel involves the following steps:

1. Identify cells to be used for decision variables. For an example, see the top of Figure 1. These are the values that Excel Solver is used to determine.
2. Compute the total cost as a function of decision variable quantities. Typically objective function coefficients are entered and the sum of the products of quantities and their corresponding unit costs are added to obtain the total cost (e.g., using the SUMPRODUCT Excel function). For an example, see the center of Figure 1, where total cost is the sum of the element-wise product of quantities and unit costs.
3. Compute the left-hand side of constraint equations. Typically constraint coefficients are entered and the sum of the products of quantities and their corresponding coefficients are added to obtain the total (e.g., using the SUMPRODUCT Excel function). For example, see the bottom section of Figure 1, where the LHS value is the sum of the element-wise product of the quantity and the constraint equation coefficients.
4. List the constraint equal or inequality sign for each constraint. This is not used by Solver, but is a cue for correctly entering constraints in Solver. For example see the bottom section of Figure 1 between the LHS column and the RHS column.
5. Enter the right hand side of constraint equations. For example, see the far right column of the bottom section of Figure 1.
6. Activate the Excel Solver following the menu "Tools > Add-Ins" and check "Solver Add-in."
7. Run Solver using "Tools > Solver…." The Solver Parameters window will appear. Figure 5 shows an example of the Solver Parameters window.
8. Select the cell with the total cost for the objective ("Set Target Cell" in the Solver Parameters window),
9. Select Min for "Equal To" to minimize the target cell.

10. Select the quantity array for the decision variables ("By Changing Cells" in the Solver Parameters window),
11. Sequentially add constraints ("Subject to the Constraints" in the Solver Parameters window) by entering the left-hand side cell(s), selecting the appropriate relationship symbol (equality or inequality), and entering the right-hand side cell(s).
12. Under "Options," select "Assume Non-Negative" to enforce the non-negativity constraint.
13. Click on Solve.

When successful, Excel will automatically update the quantities with values that satisfy the constraints and minimize the total cost of the mix.

Fig. 5 – Excel Solver Parameters Window Corresponding to Figure 3



Solver Parameters

Set Target Cell: $N$61

Equal To: ○ Max ● Min ○ Value of: 0

By Changing Cells: $C$58:$M$58

Subject to the Constraints:

$N$64 = $P$64
$N$65 <= $P$65
$N$66 <= $P$66
$N$67:$N$75 = $P$67:$P$75