Beam Deflection Using Spreadsheet Tools for Successive Integration

Shannon K. Sweeney
sks9@psu.edu
School of Engineering
Penn State Erie, The Behrend College

Abstract

This paper describes an assignment for students in a mechanics of materials course in an engineering or engineering technology program. The assignment is uniquely challenging in that it requires students to determine the deflection of a beam having a cross-section that varies continuously along its length. The assignment fulfills the need of utilizing common computer tools [1] to do realistic problems and requires students to draw upon their mathematics background.

The assignment has three purposes. The first purpose is to have students use a spreadsheet to execute the trapezoidal rule as an estimate of integration operations. The second purpose is for students to utilize the solver and/or goal seek tools of a spreadsheet tool such as EXCEL to evaluate unknowns such as the constants of integration from appropriate boundary conditions. The third purpose is for students to develop a better understanding of the relationships between internal moment, cross-section moment of inertia, beam slope, and beam deflection.

I. Introduction

As students study beam deflections in engineering and in engineering technology, they are introduced to the method of successive integration which starts with the following general equation [2][3][4].

$$\frac{M}{EI}(x) = \frac{d^2 y}{dx^2}$$

Depending upon external loading changes, material changes, and/or cross-section changes along the length of the beam, a given beam may require the development of multiple functions for *M/EI(x)*. However, for most assigned work, the function(s) for *M/EI* are simple polynomials in x that are easily integrated, first for slope θ by

$$\theta(x) = \frac{dy}{dx} = \int \frac{M}{EI}(x)dx$$

and then again for deflection y by

$$y(x) = \int \theta(x)dx\,.$$

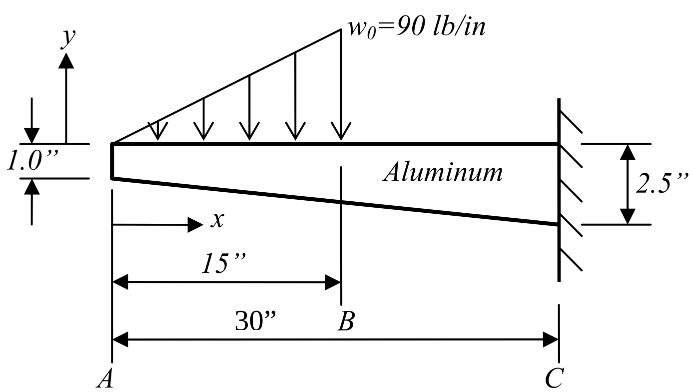
With each integration of each function, appropriate boundary conditions for slope and/or deflection must be considered to evaluate the resulting constants of integration C_1, C_2, etc. Various computer tools and simulations [5] have been developed to assist learning of beam deflections.

II. The Problem Assignment

This assignment requires students to determine deflection *y(x)* for a beam that has a linearly distributed external load and a cross-section that varies continuously along its length. With the varying cross-section, the standard beam deflection diagrams and the principle of superposition cannot be used. A tabulation of *y(x)* is acceptable from the students and a function for *y(x)* is not necessary. The material of the beam is aluminum along the entire length with modulus of elasticity $E = 10\text{x}10^6$ psi. The weight of the beam is neglected. The width b of the beam is constant at 2 inches for the entire length. The cross-section is rectangular in shape where the depth d varies linearly from 1.0" at the left end to 2.5" at the right end. Consequently, the moment of inertia *I(x)* is not a constant and can be expressed by the following equation where *I(x)* is in^4 and x is inches.

$$I_{AC}(0 \le x \le 30) = \frac{bd^3}{12} = \frac{1}{48000}\left(x^3 + 60x^2 + 1200x + 8000\right)$$

The standard assignment can be made as shown below by making the right support a fixed connection with no left support and treating the member as a cantilever beam with associated boundary conditions. A variation of the standard assignment can be made by making the beam simply supported at each end rather than a cantilever beam. An advanced assignment can be made by keeping the right support a fixed connection and adding a simple support to the left end of the beam, making it a statically indeterminate problem.



III. Standard Problem

As a cantilever beam, equilibrium analysis of the beam shows that the internal moment *M(x)* can be expressed as the following equations where *M(x)* is lb-in and x is inches.

$$M_{AB}(0 \le x \le 15) = -x^3$$

$$M_{BC}(15 \le x \le 30) = -675x + 6750$$

Substituting E, *M(x)*, and *I(x)* into the general equation above yields the following equations which can be integrated twice to obtain *y(x)*. The units of *M/EI* will be in^{-1} and x is inches.

$$\frac{M}{EI}_{AB}(0 \le x \le 15) = -\frac{3}{625} \cdot \frac{x^3}{x^3 + 60x^2 + 1200x + 8000} = \frac{d^2 y}{dx^2}$$
$$\frac{M}{EI}_{BC}(15 \le x \le 30) = -\frac{81}{25} \cdot \frac{x - 10}{x^3 + 60x^2 + 1200x + 8000} = \frac{d^2 y}{dx^2}$$

Integrating the function for $(M/EI)_{AB}$, to yield a function for slope, is difficult to do and requires many steps [6] [7]. The first step is to convert the improper fraction into the sum of a polynomial and a proper fraction. The polynomial is easily integrated but another step is needed to get the proper fraction into a form which can be integrated. The function for $(M/EI)_{BC}$ starts as a proper polynomial fraction. So, the next step for both *M/EI* functions is the procedure of resolution whereby the method of partial fractions is used in conjunction with the method of undetermined coefficients to transform the proper fraction into a sum of simpler expressions. The method of partial fractions requires that the denominator of the partial fraction be factored which is simple for this case but may be difficult for other cases. The next step is to integrate the resulting simpler expressions which can usually be accomplished with integration by parts or with the use of appropriate reduction formulas. Due diligence yields the following functions for slope.

$$\theta_{AB}(0 \le x \le 15) = -\frac{3}{625}\left(x - 60\ln(x+20) - \frac{1200}{x+20} + \frac{4000}{(x+20)^2}\right) + C_1$$
$$\theta_{BC}(15 \le x \le 30) = \frac{81(x+5)}{25(x+20)^2} + C_2$$

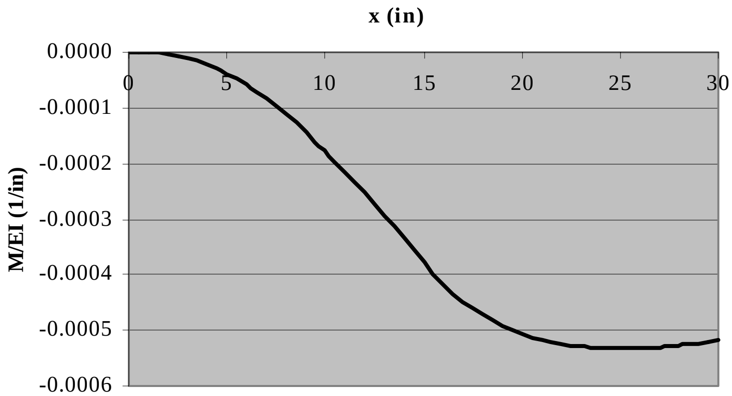
The boundary condition θ_{BC} (x = 30) = 0 can be used to show C_2 = -0.045360 and then the boundary condition θ_{BC} (x = 15) = θ_{AB} (x = 15) can be used to show C_1 =-1.093300. Integrating the slope functions, which yield functions for deflection, and evaluating the additional constants of integration will also be difficult to do.

Engineering technology students, and possibly engineering students, at the level of the first mechanics of materials course may not have been exposed to all of the steps necessary to integrate the above functions twice to find functions for $y(x)$. Even if students are later exposed to these steps, the steps are probably not retained after graduation. However, the above function results from a realistic application similar to what graduates are likely to encounter in practice.

Some students may have the ability to build code that will numerically integrate the function or some students may be able to use commercially available mathematics packages to find functions for the integration steps. However, any such work yields the same results as a simple spreadsheet and requires at least as much work as a simple spreadsheet.

Students, and especially graduates, should know that the trapezoidal rule or Simpson's rule can be used as an approximation of the definite integral for any mathematical function [8] [9] and that these rules are easily executed with a simple spreadsheet. A plot of *M/EI* vs. x as shown below indicates that, for an arbitrary cell width δ of 0.5 in, the irregular side of each cell is fairly straight meaning that the trapezoidal rule given below is appropriate.

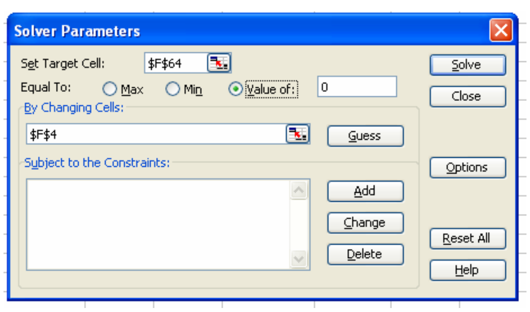
$$A = \frac{\delta}{2}\left[h_0 + h_n + 2\sum_{i=1}^{n-1} h_i\right]$$



IV. Discussion of Standard Problem

Students are encouraged to vary *x* from zero to 30 inches in 0.5 inch increments and establish this as their first spreadsheet column. Students then determine *M/EI* for each *x* value and build spreadsheet column(s) as necessary. The *M/EI* quantity becomes *h* in the trapezoidal rule for the first integration step which yields slope θ. Students should be readily capable of coding the trapezoidal rule into the spreadsheet cells. Upon completion of the trapezoidal rule, boundary conditions must be carefully considered.

During this first integration step, which yields slope, constant(s) of integration will be generated which can be evaluated from known boundary condition(s) for slope. Due to the fixed connection at the right end, the slope there will be zero so this becomes a known boundary condition for slope. The solver tool of an EXCEL spreadsheet [10], with the default options, can be used to find the slope at the left end that makes the slope zero at the right end. The corresponding solver parameters window is shown below. The slope at the right end, F64, becomes the target cell which is set equal to zero. The slope at the left end, F4, becomes the cell to be changed by the solver. There are no constraints. The goal seek tool of the spreadsheet [10] can also be used for this task.



Students can proceed to code the trapezoidal rule into the spreadsheet cells for the second integration step. The slope θ quantity then becomes *h* in the trapezoidal rule for the second integration step which yields deflection *y*.

During the second integration step, which yields deflection, additional constant(s) of integration will be generated which can be evaluated from known boundary condition(s) for deflection. Due to the fixed connection at the right end, the deflection there will be zero so this becomes a known boundary condition for deflection. The solver tool of the spreadsheet can be used to find the deflection at the left end that makes the deflection zero at the right end. Again, the solver parameters are simple and the default options are adequate. The goal seek tool of the spreadsheet can also be used for this task.

The completed spreadsheet solution of slope and deflection is provided in the Appendix. To minimize student error in the procedure, specific values of slope, deflection etc. may be provided to the students so they can check their work. The completed spreadsheet in the Appendix has separate columns for *M*, *E*, and *I* in addition to the combined *M/EI* quantity. Graphs of *M/EI(x), θ (x), y(x)*, etc. can be generated from the solution. A comparison to the exact solution shows that for deflection *y(x)*, the trapezoidal rule results in an error less than 1% for $0 < x < 25$ inches. The percentage error increases greatly for $25 < x < 30$ inches.

Successive integration to get slope and deflection functions for this problem having more than one *M/EI* function would require the use of the boundary, or continuity, conditions that slopes and deflections are equal at the point(s) of function transition to solve for the constants of integration. Such work often involves a solution to simultaneous equations to obtain the constants of integration or the use of singularity functions. The use of the trapezoidal rule and a spreadsheet as an approximation to the definite integral avoids such cumbersome mathematics.

A similar assignment can be made for a simply supported beam rather than a cantilever beam in which case there will be no known boundary conditions for slope and two known boundary conditions for deflection. For a simply supported beam, the moment equation will change according to equilibrium analysis and the slope at the left end of the beam can be assigned a dummy value for use with the trapezoidal rule. Then, the solver tool of the spreadsheet can be used to find the slope of the left end of the beam that makes the deflection at the right end of the beam equal to zero. The right-end deflection becomes the target cell which is set equal to zero. The left-end slope becomes the cell to be changed by the solver. The left-end deflection is set equal to zero as a constraint for the solver. The goal seek tool of the spreadsheet can also be used for this task.

V. Advanced Problem

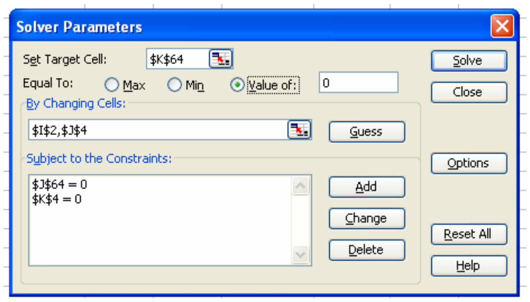
A similar assignment can also be made for statically indeterminate beams. If, in the cantilever beam problem, a simple support is added to the left end of the beam, there will be a vertical force reaction R_A at the support, making all the reactions statically indeterminate. The internal moment then becomes a function of location and the unknown reaction R_A. Substituting *E*, *M(x)*, and *I(x)* into the general equation above yields the following equations which can be integrated twice to obtain *y(x)*. The units of *M/EI* will be in^{-1} and *x* is inches.

$$\frac{M}{EI}_{AB}(0 \le x \le 15) = \frac{3}{625} \cdot \frac{-x^3 + R_A x}{x^3 + 60x^2 + 1200x + 8000} = \frac{d^2 y}{dx^2}$$

$$\frac{M}{EI}_{BC}(15 \le x \le 30) = \frac{3}{625} \cdot \frac{(R_A - 675)x + 6750}{x^3 + 60x^2 + 1200x + 8000} = \frac{d^2 y}{dx^2}$$

Integrating the functions for *M/EI*, to yield functions for slope, is again difficult to do and requires many steps. The reaction R_A would need to be treated as an unknown in the integration process, adding more complexity to the problem. For any statically indeterminate problem, a reaction force or reaction moment will be unknown until the integrations are carried out to the point where boundary conditions for slope and/or deflection can be used to evaluate unknowns. As a result, two dummy values, or unknowns, exist that need to be adjusted simultaneously to satisfy boundary conditions.

For this problem the unknowns are the magnitude of R_A and the slope of the beam at the left end. The boundary conditions are zero slope at the right end of the beam and zero deflection at both the right and left ends of the beam. The corresponding solver parameters window is shown below.



The deflection at the right end, K64, becomes the target cell which is set equal to zero. The magnitude of R_A, I2, and the slope of the beam at the left end, J4, become the cells to be changed by the solver. The deflection at the left end, K4, is set equal to zero as a constraint for the solver. The slope at the right end, J64, is set equal to zero as another constraint for the solver. Since there are two unknowns, the goal seek tool of the spreadsheet cannot be used. The completed spreadsheet solution of slope and deflection is provided in the Appendix. Graphs of *M/EI(x), θ (x), y(x)*, etc. can be generated from the solution. Beam normal stresses can then be determined from the tabulated internal moment.

VI. Evaluation and Assessment

The assignment can be used as a direct assessment of TAC of ABET, Criterion 2, Program Outcomes *a* and *f*. Outcome *a* states that graduates must demonstrate an appropriate mastery of the knowledge, techniques, skills and modern tools of their disciplines and Outcome *f* states that graduates must demonstrate an ability to identify, analyze and solve technical problems. The assignment is considered a project and, as such, constitutes up to 5% of the grade in a 3-credit course.

Student learning may be assessed on a scale of 1 to 5, defined as follows:

1 unable to initiate methods to solve assigned problem
2 initiates methods incorrectly
3 initiates methods correctly but with substantial help
4 applies correct procedures but obtains incorrect answers
5 successfully applies methods and achieves correct answers

Such numerical data and trends are not yet available but following are some assessment observations from an initial assignment of the project.

The topic of successive integration for beam deflection is often a student's first experience with an engineering application of integral calculus. The topic and the assignment require students to conceptualize and develop a plan of work. Students often display some apprehension of the assignment and may need some guidance in getting started. The instructor may need to discuss the conceptualization and planning necessary for various stages of the assignment.

Better students have said they didn't have all the mathematics skills necessary to do the integration at the time of the assignment. Due to the complexity of the function(s) to be integrated, students seem to appreciate the value of the trapezoidal rule for integration. Students also realized a substantial reduction of the computation-intensive part of the project by not having to use boundary and continuity conditions or singularity functions. Students may have more of an appreciation for the spreadsheet approach if it is assigned after they have been exposed to the mathematics necessary to do the closed-form solution and after they have had to mathematically utilize boundary and continuity conditions to evaluate several constants of integration simultaneously within a beam deflection problem.

Other computer tools, more advanced than a typical spreadsheet, may be used for the assignment. However students are encouraged to use a typical spreadsheet due to its widespread use in industry practice. Most students choose to use a spreadsheet for the entire solution although some students use a hybrid approach by finding the exact integral function, by hand or with another computer tool, and then using a spreadsheet solver to evaluate the constants of integration. Generally, there seems to be a shorter learning curve for spreadsheets than other computer tools. However, differing levels of spreadsheet proficiency among the students did hinder the conceptualization and planning stages of the process. The assignment shifts the burden of necessary background knowledge from proficiency in computation-intensive work to proficiency in computer tool (spreadsheet) utilization. Curriculum offerings must align accordingly.

VII. Conclusion

The assignment can be made with many variations to the distributed load, the beam cross-section, etc. The assignment provides students with an applicable and easily conceptualized problem which results in a mathematical function that is not easily integrated twice. The assignment shows students the utility of estimation tools such as the trapezoidal rule for functions that are not easily integrated. The assignment forces students to use appropriate boundary conditions to determine constants of integration through various computer tools. Throughout the assignment, students develop a better understanding of the relationships between internal moment, cross-section moment of inertia, beam slope, and beam deflection.

The assignment provides an opportunity to demonstrate how all coursework may be related within a given curriculum. The trapezoidal rule is covered in a previous technical mathematics course, use of Excel and its tools is covered in a previous computer applications course, and the engineering mechanics are covered in the current mechanics of materials course. Although not often popular among students, an early realization of coursework dependency will translate to successful professionals. Overall, the assignment provides a focus on solving a problem while not losing touch with the underlying theory and it enhances the relevance of a course topic to a realistic situation.

References

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APPENDIX

General Data			Cantilever Beam Problem				Statically Indeterminate Problem R_A (lb) = 295.9			
x (in)	E (lb/in^2)	I (in^4)	M (lb-in)	M/EI (1/in)	θ (rad)	y (in)	M (lb-in)	M/EI (1/in)	θ (rad)	y (in)
0.0	10000000	0.1667	0	0.00000000	0.009471	-0.19358	0	0.00000000	-0.003310	0.00000
0.5	10000000	0.1795	0	-0.00000007	0.009471	-0.18884	148	0.00008237	-0.003289	-0.00165
1.0	10000000	0.1929	-1	-0.00000052	0.009470	-0.18411	295	0.00015286	-0.003230	-0.00328
1.5	10000000	0.2070	-3	-0.00000163	0.009470	-0.17937	441	0.00021276	-0.003139	-0.00487
2.0	10000000	0.2218	-8	-0.00000361	0.009469	-0.17464	584	0.00026320	-0.003020	-0.00641
2.5	10000000	0.2373	-16	-0.00000658	0.009466	-0.16990	724	0.00030518	-0.002878	-0.00789
3.0	10000000	0.2535	-27	-0.00001065	0.009462	-0.16517	861	0.00033959	-0.002717	-0.00928
3.5	10000000	0.2704	-43	-0.00001586	0.009455	-0.16044	993	0.00036723	-0.002540	-0.01060
4.0	10000000	0.2880	-64	-0.00002222	0.009446	-0.15572	1120	0.00038879	-0.002351	-0.01182
4.5	10000000	0.3064	-91	-0.00002974	0.009433	-0.15100	1241	0.00040491	-0.002153	-0.01295
5.0	10000000	0.3255	-125	-0.00003840	0.009416	-0.14629	1355	0.00041615	-0.001947	-0.01397
5.5	10000000	0.3454	-166	-0.00004816	0.009394	-0.14158	1461	0.00042300	-0.001737	-0.01489
6.0	10000000	0.3662	-216	-0.00005899	0.009367	-0.13689	1560	0.00042592	-0.001525	-0.01571
6.5	10000000	0.3877	-275	-0.00007083	0.009335	-0.13222	1649	0.00042531	-0.001312	-0.01642
7.0	10000000	0.4101	-343	-0.00008365	0.009296	-0.12756	1729	0.00042152	-0.001101	-0.01702
7.5	10000000	0.4333	-422	-0.00009737	0.009251	-0.12292	1798	0.00041489	-0.000892	-0.01752
8.0	10000000	0.4573	-512	-0.00011195	0.009198	-0.11831	1855	0.00040571	-0.000686	-0.01791
8.5	10000000	0.4823	-614	-0.00012734	0.009139	-0.11373	1901	0.00039423	-0.000486	-0.01821
9.0	10000000	0.5081	-729	-0.00014347	0.009071	-0.10917	1934	0.00038070	-0.000293	-0.01840
9.5	10000000	0.5348	-857	-0.00016030	0.008995	-0.10466	1954	0.00036533	-0.000106	-0.01850
10.0	10000000	0.5625	-1000	-0.00017778	0.008910	-0.10018	1959	0.00034832	0.000072	-0.01851
10.5	10000000	0.5911	-1158	-0.00019584	0.008817	-0.09575	1950	0.00032983	0.000242	-0.01843
11.0	10000000	0.6206	-1331	-0.00021445	0.008714	-0.09137	1924	0.00031004	0.000402	-0.01827
11.5	10000000	0.6512	-1521	-0.00023356	0.008602	-0.08704	1882	0.00028907	0.000551	-0.01803
12.0	10000000	0.6827	-1728	-0.00025313	0.008481	-0.08277	1823	0.00026706	0.000690	-0.01772
12.5	10000000	0.7152	-1953	-0.00027310	0.008349	-0.07856	1746	0.00024414	0.000818	-0.01735
13.0	10000000	0.7487	-2197	-0.00029345	0.008208	-0.07442	1650	0.00022040	0.000934	-0.01691
13.5	10000000	0.7832	-2460	-0.00031413	0.008056	-0.07035	1535	0.00019594	0.001039	-0.01641
14.0	10000000	0.8188	-2744	-0.00033511	0.007893	-0.06637	1399	0.00017085	0.001130	-0.01587
14.5	10000000	0.8555	-3049	-0.00035636	0.007721	-0.06246	1242	0.00014522	0.001209	-0.01529
15.0	10000000	0.8932	-3375	-0.00037784	0.007537	-0.05865	1064	0.00011911	0.001275	-0.01467
15.5	10000000	0.9321	-3713	-0.00039831	0.007343	-0.05493	874	0.00009381	0.001329	-0.01401
16.0	10000000	0.9720	-4050	-0.00041667	0.007139	-0.05131	685	0.00007046	0.001370	-0.01334
16.5	10000000	1.0131	-4388	-0.00043309	0.006927	-0.04779	495	0.00004889	0.001399	-0.01265
17.0	10000000	1.0553	-4725	-0.00044775	0.006707	-0.04438	306	0.00002898	0.001419	-0.01194
17.5	10000000	1.0986	-5063	-0.00046080	0.006479	-0.04109	116	0.00001058	0.001429	-0.01123
18.0	10000000	1.1432	-5400	-0.00047237	0.006246	-0.03791	-73	-0.00000641	0.001430	-0.01052
18.5	10000000	1.1889	-5738	-0.00048259	0.006007	-0.03484	-263	-0.00002211	0.001423	-0.00980
19.0	10000000	1.2358	-6075	-0.00049158	0.005764	-0.03190	-452	-0.00003660	0.001408	-0.00910
19.5	10000000	1.2840	-6413	-0.00049943	0.005516	-0.02908	-642	-0.00004999	0.001386	-0.00840
20.0	10000000	1.3333	-6750	-0.00050625	0.005265	-0.02638	-831	-0.00006236	0.001358	-0.00771
20.5	10000000	1.3840	-7088	-0.00051212	0.005010	-0.02382	-1021	-0.00007377	0.001324	-0.00704
21.0	10000000	1.4359	-7425	-0.00051711	0.004753	-0.02138	-1210	-0.00008430	0.001285	-0.00639
21.5	10000000	1.4890	-7763	-0.00052131	0.004493	-0.01906	-1400	-0.00009402	0.001240	-0.00576
22.0	10000000	1.5435	-8100	-0.00052478	0.004232	-0.01688	-1590	-0.00010298	0.001191	-0.00515
22.5	10000000	1.5993	-8438	-0.00052758	0.003969	-0.01483	-1779	-0.00011124	0.001137	-0.00457
23.0	10000000	1.6564	-8775	-0.00052976	0.003704	-0.01291	-1969	-0.00011885	0.001080	-0.00401
23.5	10000000	1.7149	-9113	-0.00053139	0.003439	-0.01113	-2158	-0.00012585	0.001019	-0.00349
24.0	10000000	1.7747	-9450	-0.00053249	0.003173	-0.00948	-2348	-0.00013229	0.000954	-0.00300
24.5	10000000	1.8359	-9788	-0.00053313	0.002907	-0.00796	-2537	-0.00013820	0.000887	-0.00253
25.0	10000000	1.8984	-10125	-0.00053333	0.002640	-0.00657	-2727	-0.00014363	0.000816	-0.00211
25.5	10000000	1.9624	-10463	-0.00053314	0.002373	-0.00532	-2916	-0.00014861	0.000743	-0.00172
26.0	10000000	2.0278	-10800	-0.00053259	0.002107	-0.00420	-3106	-0.00015316	0.000668	-0.00137
26.5	10000000	2.0947	-11138	-0.00053170	0.001841	-0.00321	-3295	-0.00015732	0.000590	-0.00105
27.0	10000000	2.1630	-11475	-0.00053052	0.001575	-0.00235	-3485	-0.00016112	0.000510	-0.00078
27.5	10000000	2.2327	-11813	-0.00052906	0.001310	-0.00163	-3674	-0.00016457	0.000429	-0.00054
28.0	10000000	2.3040	-12150	-0.00052734	0.001046	-0.00104	-3864	-0.00016771	0.000346	-0.00035
28.5	10000000	2.3768	-12488	-0.00052540	0.000783	-0.00059	-4054	-0.00017055	0.000261	-0.00020
29.0	10000000	2.4510	-12825	-0.00052325	0.000521	-0.00026	-4243	-0.00017311	0.000175	-0.00009
29.5	10000000	2.5268	-13163	-0.00052091	0.000260	-0.00007	-4433	-0.00017542	0.000088	-0.00002
30.0	10000000	2.6042	-13500	-0.00051840	0.000000	0.00000	-4622	-0.00017749	0.000000	0.00000