EFFECTIVE OR RMS VOLTAGE OF A SINUSOID

by

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In most, if not all, engineering technology electric circuit books, we are told that the effective or root-mean-square (*rms*) voltage of a sinusoid is obtained from the peak voltage through a formula that can be derived "using calculus." They mention of course that the *rms* value is the equivalent dc voltage that would produce the same average power. This Tech Tip illustrates another technique for obtaining the relationship between *rms* and peak voltage through the use of the power formula and a trigonometric identity.

Let us consider a resistor R connected across an ac source given by

$$v = V_p \sin \omega t$$

The instantaneous power will then be

$$p = \frac{v^2}{R} = \frac{V_p^2}{R} \sin^2 \omega t$$

Using the trigonometric identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, we write

$$p = \frac{V_p^2}{2R}(1 - \cos 2\omega t) = \frac{V_p^2}{2R} - \frac{V_p^2}{2R} \cos 2\omega t$$

Clearly, the average power of this expression is $V_p^2/2R$ since the average value of the cosine wave is zero; in other words, it is the "dc" portion of the waveform. To better see this, if we make a plot of the power versus time for V_p = 1 V, R = 1 Ω and f = 1 kHz, as shown in Figure 1, we can see that it is an inverted cosine wave with twice the frequency and oscillating between zero and 1 W. The center of the oscillation is 0.5 W, which is the average value. As expected, the instantaneous power is always positive and is zero at the zero-crossing points of the sinusoidal voltage.

Equating the power delivered by a dc source to the average power delivered by the ac source using the same resistor *R*, we write

$$\frac{V_{dc}^2}{R} = \frac{V_p^2}{2R}$$

Canceling *R* and getting the square root of both sides, we obtain the known relationship

$$V_{dc} = \frac{1}{\sqrt{2}} V_p = 0.707 V_p = V_{rms}$$

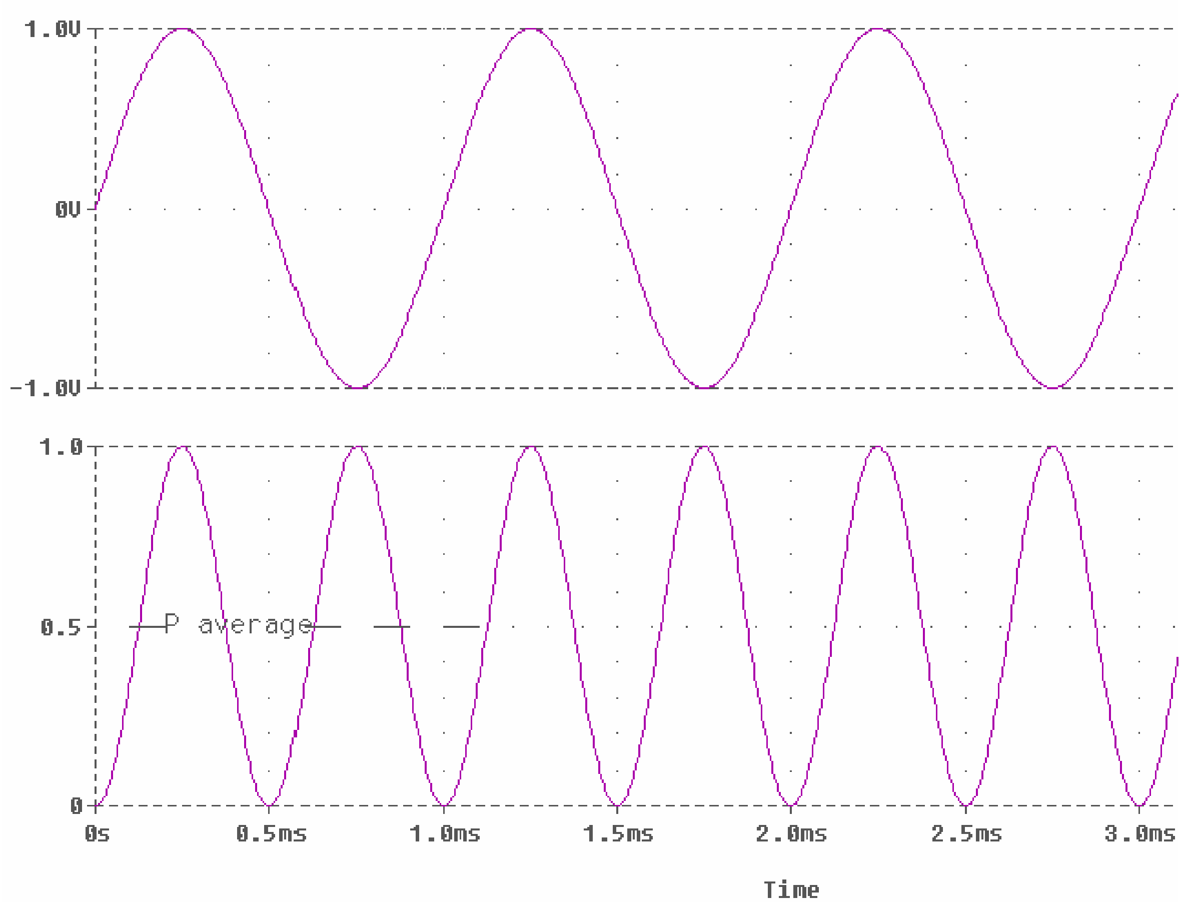

Figure 1. Voltage and power for V_p = 1 V, $R = 1\Omega$, and f = 1 kHz