A Dynamics Project: Quarter-Mile Time of a Dragster

Shannon Sweeney
sks9@psu.edu
School of Engineering and Engineering Technology
Penn State Erie, The Behrend College

ABSTRACT

This paper describes an applied assignment for undergraduate students in a dynamics course. The assignment relates to an interest of many engineering and technology students: drag racing. Students are given enough information about a popular sports car model, and some simplifying assumptions, to estimate its time in the quarter-mile. Students can then compare their results to published drag racing results for that vehicle. Even with the simplifying assumptions, the calculated results are strikingly close to the published results.

INTRODUCTION

The purpose of this project was to make a comprehensive assignment that is real and interesting to engineering and technology students. Many assignments in a dynamics course are strictly academic in nature, not attracting the students' interest and often leaving the students wondering about the application. Most engineering and technology students have at least a cursory interest in drag racing and some may be full-fledged enthusiasts or participants. Consequently, this assignment should have broad appeal in a dynamics course.

Students are given measured engine performance data for a Chevrolet Corvette C6-05. They are given the approximate values for the weight of the car, its transmission shift schedule, its differential ratio, its tire diameter, and a common method for estimating both rolling resistance and wind resistance. Students are also given the actual drag racing results published for a 1999 Chevrolet Corvette Coupe. It is not known if the weight, transmission, differential, and tire data given to the students matches that of the car for which race data is published but they are assumed to be close enough for the purpose of this assignment.

The assignment is comprehensive in that it requires utilizing the relationship of power to force and acceleration in free-body and kinetic diagrams, requires modeling of dynamic resistive forces, and requires finding displacement and time associated with non-constant acceleration. The five race data that are published include 1/4 Mile ET (elapsed time), 1/4 Mile MPH (miles per hour), 1/8 Mile ET, 1/8 Mile MPH, and 0-60 Foot ET. By carefully establishing a spreadsheet to carry out the appropriate integrations, students should able to closely match all five race data.

THE MODEL

Free-Body and Kinetic Diagrams

By modeling the car as a particle and by equating free-body and kinetic diagrams, students should be able to show:

$$a = (1/m)(P/v - F_R - F_W)$$

where a = instantaneous acceleration of the car,
m = mass of the car (students are given a *weight* W of 3200 lb),
P = instantaneous power (from engine performance data),
v = instantaneous velocity of the car,
F_R = rolling resistance (later assumed constant), and
F_W = wind resistance (later assumed proportional to square of velocity).

Engine Performance Data

Students are given the following tabulation of engine speed and wheel horsepower which is available from RRI [1]. Full-throttle operation is assumed for the curve.

Engine Speed n (rpm)	Wheel Power P (hp)
1751	100.6
1993	118.3
2493	151.3
2994	186.7
3496	219.9
3999	267.8
4214	281.3
4401	295.9
4603	309.0
5006	331.3
5507	346.0
5810	353.9
6010	353.8
6212	350.2
6429	346.6

The tests conducted by RRI determine wheel horsepower which is the power available at the wheel to propel the vehicle forward. By using wheel horsepower, drivetrain inefficiencies have been accounted for.

Transmission Shift Schedule

Students are given the following transmission shift-up schedule:

Gear	Ratio	Start rpm	Shift rpm
1	3.27	1700	5500
2	2.20	3700	5500
3	1.56	3900	5500
4	1.22	4300	5500
5	1.00	4500	5500
6	0.82	4500	~

The ratio is transmission input (engine output) speed to transmission output speed.

Differential Ratio and Tire Diameter

Students are given a differential ratio of 3.08. This ratio is differential input (transmission output) speed to differential output (wheel input) speed. Students are given a tire diameter of 25.66 inches.

Rolling Resistance

Rolling resistance is simply modeled with a coefficient of rolling friction f_r [2]. A value of 0.012 is given for f_r which is a commonly assumed value for pneumatic rubber tires on the hard surface of a drag strip. F_R is then equal to $f_r W$ and students should be able to show that it is constant at 38.4 lb.

Wind Resistance

Wind resistance is modeled as a quadratic loss or a velocity-squared loss [3]. Students are given the following relationship and values:

$$F_W = (C\rho A/2)v^2$$

where F_W = wind resistance,
C = drag coefficient (≈ 0.35 for sleek sports cars),
ρ = fluid (air) density (accepted as 0.0807 lb_m/ft^3 @ STP),
A = frontal area (≈ 24 ft^2; based on the approximate width and height), and
v = velocity of the car.

The biggest challenge for students will be proper handling of units. For the given information, students should be able to show:

$$F_W = 0.01053v^2$$

where F_W = wind resistance force in lb,
v = instantaneous velocity of car in ft/s,
and the units for the constant, 0.01053, are $lb\text{-}s^2/ft^2$.

Simplifying Assumptions

Some simplifying assumptions stated for the students are:

1) Engine performance data, although produced during steady-state conditions, applies to acceleration conditions.
2) Automatic transmission and neglect shift times.
3) Torque converter locked up at all times (no torque multiplication).
4) Neglect mass moments of inertia of wheels, drivetrain, and engine.
5) Sufficient tire grip coefficient to prevent slip.
6) Neglect tire growth.
7) Assume a velocity of 12.9 mph at start (engine at 1700 rpm with transmission in first gear). The actual practice of starting a race usually involves running the engine to about 4000 rpm with the brakes locked and the torque converter stalled, and then releasing the brakes. Students at the freshman or sophomore level will not have the background to model the behavior of the torque converter and its effect on acceleration. Therefore the assumption of a 12.9 mph head start approximates the actual practice when starting a race.

The simplifying assumptions mean that engine speed n and car velocity v will be directly proportional, with the proportionality depending on the transmission gear.

THE ANALYSIS

With the given data and simplifying assumptions, students should be able to show:

$$a = 0.01006(P/v - 38.4 - 0.01053v^2)$$

where a = instantaneous acceleration of car in ft/s^2,
P = instantaneous wheel power in ft-lb/s,
v = instantaneous velocity of car in ft/s,
and the units for the constant, 0.01006, are lb-s^2/ft or slugs.

Wheel power P will be a function of car velocity related through the engine data, transmission gear, differential ratio and tire diameter. Consequently, car acceleration a will be non-constant and it will be a determinable function of car velocity v.

Initially, it may appear to be advantageous to make car velocity v the controlled variable throughout the project and find car acceleration a as a function of car velocity. However, transmission shifts are based on engine speed, not car velocity. As a result, the author has found it advantageous to make engine speed the controlled variable throughout the project and find car acceleration a and car velocity v as a function of engine speed n. Either approach must utilize the assumption that engine speed and car velocity are directly proportional.

With engine speed being the controlled variable throughout the project, students will need wheel power expressed as a function of engine speed. This can be accomplished several ways but students are encouraged to develop an equation for wheel power as a function of engine speed from the provided tabulated engine data. Students should find that a third-order polynomial as follows fits quite well as shown in Figure 1.

$P = -4.04e\text{-}9n^3 + 4.08e\text{-}5n^2 - 0.0567n + 99.2$

where P = *wheel* horsepower, and

n = engine rpm (revolutions per minute).

This equation has a coefficient of determination r^2 of 0.9993, meaning that it can be used to accurately determine wheel power from engine speed in the range: 1700 rpm < n < 6500 rpm.

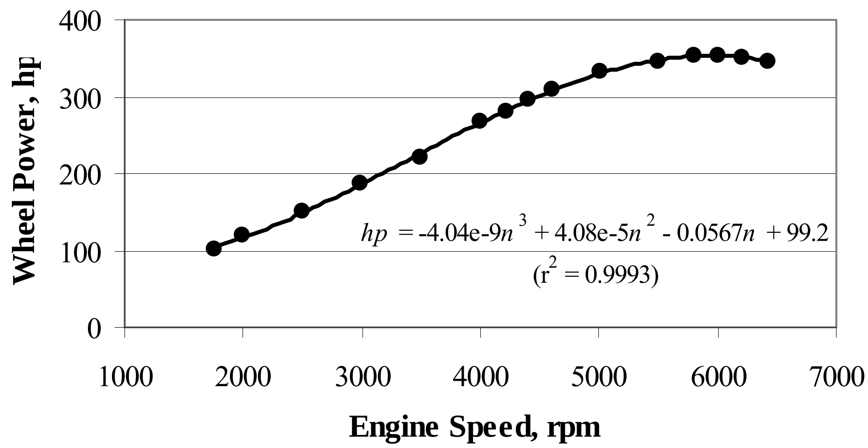

Figure 1. Engine Performance Data

With engine speed as the controlled variable, students will need to express the car velocity as a function of engine speed. The function is determined by the transmission gear, differential ratio, and tire diameter. Figure 2 shows the relationships students will need to develop and utilize to express v as a function of n.

Gear	n/v (engine rpm/ car ft/s)	dv (ft/s) (dn = 200 rpm)
1	89.96	2.223
2	60.52	3.305
3	42.92	4.660
4	33.56	5.959
5	27.51	7.270
6	22.56	8.865

Figure 2. Relationship of Engine Speed to Car Velocity

With all things considered, the instantaneous acceleration throughout the race is a determinable function of instantaneous velocity. So, for any engine speed and each transmission gear, the instantaneous car velocity and the instantaneous car acceleration can be determined.

Since instantaneous acceleration is a function of instantaneous velocity, the following relationships will be necessary:

$a = dv/dt$, leading to an integration of $dt = (1/a)dv$

and

$a = vdv/ds$, leading to an integration of $ds = (v/a)dv$.

Continuous functions for $(1/a)$ and (v/a) can be derived from the curve-fit equation of the engine data and the information in Figure 2 but the functions are rather complicated. The functions could be integrated by closed form techniques but are best handled by a tabulation and subsequent use of the trapezoidal rule.

By using engine speed as the controlled variable throughout the project, students will need to develop a tabulated integration for time t as the engine goes from start rpm to shift rpm for each transmission gear. Students will also need to develop a tabulated integration for displacement s as the engine goes from start rpm to shift rpm for each transmission gear. From the resulting tabulation of t and s (each as a function of engine speed and transmission gear), the five race data can be determined.

SAMPLE ASSIGNMENT

Although the assignment can be given several ways, the author has chosen to give all the necessary information to the students at the beginning of the project and allow students about four weeks to complete the project. The information provided is the vehicle weight, engine performance tabulation, transmission schedule, differential ratio, tire diameter, coefficient of rolling friction, and wind resistance information which includes the drag coefficient, frontal area, air density, and the quadratic loss equation. Simplifying assumptions are also given and discussed.

To assist the students, the author has broken the project into six parts and uses a small amount of class time discussing each part, about every other class period. The first category is development of appropriate free body and kinetic diagrams.

The second part is development of the horsepower equation as a function of engine speed. Curve-fitting procedures and options within spreadsheets are discussed with the students. Students are asked to plot the data and their curve-fit as shown in Figure 1. They should find a maximum horsepower of about 350 and that it occurs at about 6000 rpm. From this, students need to realize that the stated horsepower of an engine occurs at a specific speed and that the engine does not have the stated horsepower at all speeds.

Students should establish a spreadsheet using engine speed as the controlled variable, showing start rpm to shift rpm for each transmission gear. Engine speed increments of 200 rpm are encouraged. Students may be provided a spreadsheet template with this much information so they start the project correctly and uniformly. Students must then find the wheel horsepower for each engine speed from the engine performance equation.

It should be converted to appropriate units of ft-lb/s and students are encouraged to create spreadsheet columns for power in both units.

The third part is development of the vehicle velocity in ft/s as a function of engine speed as shown in Figure 2. The roles of the transmission, differential, and tire are discussed with the students. The car velocity should be converted to familiar units of mph and students are encouraged to create spreadsheet columns for velocity in both units. Throughout this part, students should be able to consider the transmission schedule and show that the transmission output speed at shift rpm for any gear is the same as it is for the start rpm at the next higher gear.

The fourth part is modeling the force from wheel horsepower and the dynamic resistive forces for the free body diagram. The relationship of power and velocity and the subjects of rolling friction and quadratic loss wind resistance are reviewed with the students. The instantaneous values of each force can be determined as a function of engine speed.

The fifth part is development of acceleration. The instantaneous values of rolling resistance and wind resistance are used in conjunction with the wheel force and mass m ($=W/g$) to get the instantaneous acceleration in ft/s^2 as a function of engine speed. Acceleration should be converted to familiar units of g's and students are encouraged to create spreadsheet columns for acceleration in both units. Also, the necessary integration relationships are discussed, revealing the quantities $1/a$ and v/a as those to be integrated.

The sixth part is the integration of $1/a$ and v/a respectively. Students can determine the instantaneous values of $1/a$ and integrate that quantity with respect to velocity, by utilizing the trapezoidal rule, to find values of elapsed time t. Students can also determine the instantaneous values of v/a and integrate that quantity with respect to velocity, by utilizing the trapezoidal rule, to find values of distance traveled s. For each integration, and using 200 engine rpm increments, students will have to develop and utilize the values of the differential term dv shown in Figure 2. From the tabulation of t and s, the five race data can be determined. The completed spreadsheet is shown in Figure 3.

Given the amount of assistance provided in the six parts, students are made aware of the published results shown in Figure 4 upon completion of the project. With the published results, students can assess their performance in the exercise.

Quarter-Mile Time
Spreadsheet Results

Gear	n (rpm)	*P* (hp)	*P* (lb-ft/s)	*v* (ft/s)	*v* (mph)	*P/v* (lb)	F_R (lb)	F_W (lb)	*a* (ft/s^2)	*a* (g's)	*1/a* (s^2/ft)	*v/a* (s)	*t* (s)	*s* (ft)
1	1700	100.9	55480	18.9	12.9	2936	38.4	3.8	29.1	0.904	0.0343	0.649	0.00	0
1	1900	111.0	61076	21.1	14.4	2892	38.4	4.7	28.7	0.890	0.0349	0.737	0.08	2
1	2100	122.6	67454	23.3	15.9	2889	38.4	5.7	28.6	0.889	0.0349	0.815	0.15	3
1	2300	135.5	74507	25.6	17.4	2914	38.4	6.9	28.9	0.896	0.0346	0.886	0.23	5
1	2500	149.3	82129	27.8	18.9	2955	38.4	8.1	29.3	0.909	0.0342	0.950	0.31	7
1	2700	164.0	90212	30.0	20.5	3006	38.4	9.5	29.8	0.924	0.0336	1.009	0.38	9
1	2900	179.4	98652	32.2	22.0	3060	38.4	10.9	30.3	0.941	0.0330	1.064	0.46	12
1	3100	195.2	107339	34.5	23.5	3115	38.4	12.5	30.8	0.957	0.0324	1.118	0.53	14
1	3300	211.2	116169	36.7	25.0	3167	38.4	14.2	31.3	0.973	0.0319	1.171	0.60	17
1	3500	227.3	125034	38.9	26.5	3214	38.4	15.9	31.8	0.987	0.0315	1.224	0.67	19
1	3700	243.3	133828	41.1	28.0	3254	38.4	17.8	32.2	0.999	0.0311	1.278	0.74	22
1	3900	259.0	142444	43.4	29.6	3286	38.4	19.8	32.5	1.009	0.0308	1.335	0.81	25
1	4100	274.1	150775	45.6	31.1	3308	38.4	21.9	32.7	1.015	0.0306	1.395	0.88	28
1	4300	288.6	158716	47.8	32.6	3320	38.4	24.1	32.8	1.018	0.0305	1.458	0.95	31
1	4500	302.1	166158	50.0	34.1	3321	38.4	26.4	32.8	1.018	0.0305	1.527	1.01	35
1	4700	314.5	172995	52.2	35.6	3311	38.4	28.7	32.6	1.014	0.0306	1.601	1.08	38
1	4900	325.7	179122	54.5	37.1	3288	38.4	31.2	32.4	1.006	0.0309	1.682	1.15	42
1	5100	335.3	184430	56.7	38.7	3253	38.4	33.8	32.0	0.994	0.0312	1.771	1.22	45
1	5300	343.3	188814	58.9	40.2	3205	38.4	36.6	31.5	0.978	0.0318	1.871	1.29	50
1	5500	349.4	192167	61.1	41.7	3143	38.4	39.4	30.8	0.958	0.0324	1.982	1.36	54
2	3700	243.3	133828	61.1	41.7	2189	38.4	39.4	21.2	0.660	0.0471	2.878	1.36	54
2	3900	259.0	142444	64.4	43.9	2210	38.4	43.7	21.4	0.665	0.0467	3.010	1.52	64
2	4100	274.1	150775	67.8	46.2	2225	38.4	48.3	21.5	0.668	0.0465	3.148	1.67	74
2	4300	288.6	158716	71.1	48.4	2234	38.4	53.2	21.6	0.669	0.0464	3.297	1.82	84
2	4500	302.1	166158	74.4	50.7	2234	38.4	58.2	21.5	0.668	0.0465	3.457	1.98	96
2	4700	314.5	172995	77.7	53.0	2227	38.4	63.5	21.4	0.664	0.0468	3.631	2.13	107
2	4900	325.7	179122	81.0	55.2	2212	38.4	69.0	21.2	0.658	0.0472	3.823	2.29	120
2	5100	335.3	184430	84.3	57.5	2188	38.4	74.8	20.9	0.648	0.0479	4.036	2.44	133
2	5300	343.3	188814	87.6	59.7	2156	38.4	80.8	20.5	0.636	0.0488	4.274	2.60	146
2	5500	349.4	192167	90.9	62.0	2114	38.4	87.0	20.0	0.622	0.0500	4.541	2.77	161
3	3900	259.0	142444	90.9	62.0	1568	38.4	86.9	14.5	0.451	0.0689	6.262	2.77	161
3	4100	274.1	150775	95.5	65.1	1578	38.4	96.1	14.5	0.451	0.0688	6.575	3.09	191
3	4300	288.6	158716	100.2	68.3	1584	38.4	105.7	14.5	0.450	0.0690	6.914	3.41	222
3	4500	302.1	166158	104.9	71.5	1585	38.4	115.8	14.4	0.447	0.0695	7.284	3.73	255
3	4700	314.5	172995	109.5	74.7	1580	38.4	126.3	14.2	0.442	0.0702	7.691	4.06	290
3	4900	325.7	179122	114.2	77.8	1569	38.4	137.3	14.0	0.435	0.0713	8.144	4.39	327
3	5100	335.3	184430	118.8	81.0	1552	38.4	148.7	13.7	0.427	0.0728	8.652	4.72	366
3	5300	343.3	188814	123.5	84.2	1529	38.4	160.6	13.4	0.416	0.0747	9.227	5.07	408
3	5500	349.4	192167	128.2	87.4	1500	38.4	172.9	13.0	0.403	0.0771	9.886	5.42	452
4	4300	288.6	158716	128.1	87.4	1239	38.4	172.9	10.3	0.321	0.0967	12.396	5.42	452
4	4500	302.1	166158	134.1	91.4	1239	38.4	189.4	10.2	0.316	0.0983	13.178	6.00	529
4	4700	314.5	172995	140.1	95.5	1235	38.4	206.6	10.0	0.309	0.1004	14.057	6.59	610
4	4900	325.7	179122	146.0	99.6	1227	38.4	224.5	9.7	0.301	0.1031	15.057	7.20	697
4	5100	335.3	184430	152.0	103.6	1214	38.4	243.2	9.4	0.291	0.1066	16.207	7.82	790
4	5300	343.3	188814	157.9	107.7	1195	38.4	262.7	9.0	0.280	0.1111	17.549	8.47	890
4	5500	349.4	192167	163.9	111.8	1172	38.4	282.9	8.6	0.266	0.1168	19.136	9.15	1000
5	4500	302.1	166158	163.6	111.5	1016	38.4	281.7	7.0	0.217	0.1429	23.368	9.15	1000
5	4700	314.5	172995	170.8	116.5	1013	38.4	307.3	6.7	0.208	0.1490	25.461	10.21	1177
5	4900	325.7	179122	178.1	121.4	1006	38.4	334.1	6.4	0.198	0.1569	27.955	11.33	1371
5	5100	335.3	184430	185.4	126.4	995	38.4	361.9	6.0	0.186	0.1671	30.987	12.50	1586
5	5300	343.3	188814	192.7	131.4	980	38.4	390.8	5.5	0.172	0.1804	34.758	13.77	1824
5	5500	349.4	192167	199.9	136.3	961	38.4	420.9	5.1	0.157	0.1980	39.585	15.14	2095
6	4500	302.1	166158	199.5	136.0	833	38.4	419.1	3.8	0.117	0.2647	52.817	15.14	2095
6	4700	314.5	172995	208.4	142.1	830	38.4	457.2	3.4	0.105	0.2969	61.874	17.63	2603
6	4900	325.7	179122	217.2	148.1	825	38.4	496.9	2.9	0.090	0.3436	74.637	20.47	3208
6	5100	335.3	184430	226.1	154.2	816	38.4	538.3	2.4	0.075	0.4158	94.016	23.84	3956
6	5300	343.3	188814	235.0	160.2	804	38.4	581.4	1.8	0.057	0.5406	127.027	28.08	4936
6	5500	349.4	192167	243.8	166.3	788	38.4	626.1	1.2	0.039	0.8037	195.970	34.04	6368
6	5700	353.4	194382	252.7	172.3	769	38.4	672.4	0.6	0.018	1.7016	429.987	45.15	9143
6	5900	355.2	195353	261.6	178.3	747	38.4	720.4	-0.1	-0.004	~	~	~	~
6	6100	354.5	194972	270.4	184.4	721	38.4	770.1	-0.9	-0.027	~	~	~	~
6	6400	348.4	191634	283.7	193.5	675	38.4	847.7	-2.1	-0.066	~	~	~	~

Figure 3. Completed Spreadsheet with Integration

CONCLUSION

The appropriate race data can be interpolated from the tabulation of *t* vs. *s* and compared to published [4] data for a similar car. Figure 4 summarizes the results.

Race Data	Calculated	Published
1/4 Mile ET (s)	11.04	10.97
1/4 Mile MPH	120.1	124.7
1/8 Mile ET (s)	6.94	6.99
1/8 Mile MPH	97.9	96.6
0-60 Foot ET (s)	1.46	1.50

Figure 4. Comparative Race Data

From the tabulation of calculated results in Figure 3, some meaningful, realistic observations can be made. The car will reach 60 feet of travel shortly after it shifts into second gear. The car will reach an eighth of a mile, or 660 feet, while it is in fourth gear. The car will reach a quarter mile, or 1320 feet, shortly after it shifts into fifth gear.

The acceleration of the car is non-constant but varies only slightly from engine start rpm to engine shift rpm within each gear. The acceleration decreases with increasing gear. These are common observations for full-throttle acceleration in any car.

The car acceleration in first gear is about 1 g. Depending on the weight distribution and the center-of-gravity height, the front wheels will have little or no contact with the road during first gear acceleration which is a common observation for many dragsters. Consequently, most of the vehicle weight will be on the rear wheels during first gear acceleration and the tire grip coefficient will need to be about 1.0 to prevent slip, Figure 5.

Figure 5. Corvette Acceleration
Courtesy of Edgar Perez (car driver/owner) and Michael Yoskich (photographer).

The wind resistance becomes significant, especially at speeds greater than 100 mph. Even for a sleek Corvette, the wind resistance force at 100 mph is over 220 lb.

The top speed is estimated at about 175 mph which is realistic for this Corvette. At this speed, the summation of the resistance forces from rolling and wind equal or exceed the force available from the engine to propel the vehicle forward. Acceleration goes to zero because force equilibrium has been reached.

This assignment should be of great interest to most engineering and technology students. It should provide an exciting and familiar application of many dynamics principles. Consequently, students should be motivated to spend enough time on the project to develop a better understanding of various dynamics principles.

REFERENCES

1. Rototest Research Institute, Salemsvagen 20, S-1444 40 RONNINGE, SWEDEN www.rototestinstitute.org

2. Tongue, B.H. and S.D. Sheppard (2005). Dynamics – Analysis and Design of Systems in Motion. New York, NY: John Wiley & Sons

3. Inman, D.J. (2002). Engineering Vibration. Upper Saddle River, NJ: Prentice-Hall

4. DragTimes.com, P.O. Box 827051, Pembroke Pines, FL 33082-7051 www.dragtimes.com