

# Modeling and Performance Analysis of Mobile Ad Hoc Networks

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by

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**Abstract:** *In this paper, we propose a novel model to analyze the impact of mobility on the performance of a wireless mobile ad hoc network. In the proposed model, among each mobile communicating pair, one is considered to be stationary while the other is moving with a speed relative to the first one. Using this method, the system performance parameters such as the average radio range dwell time, the link holding time, the link breaking probability, and the system throughput are obtained by both theoretical framework and simulation. The results obtained from the proposed analytical model are observed to match well with simulation results.*

## I. INTRODUCTION

Recent advances in communications and networks have provided portable computers with wireless interfaces to communicate with each other over a wireless channel. The mobile computing no longer requires users to maintain a fixed and universally known position in the network and enables almost unrestricted mobility. A mobile ad hoc network (MANET) is a special type of wireless mobile network in which a collection of mobile nodes with wireless network interfaces may form a temporary network, without the aid of any established infrastructure or centralized administration. Therefore, it provides a flexible and low cost network solution in times of emergency or where the infrastructures are not provided or not feasible. Conference site, hospital, rescue, battlefields, and disaster relief activities are several typical scenarios where MANETs can be very useful.

In MANETs, the nodes are mobile and the network topology changes very frequently. Two nodes can directly communicate only when they are within each other's radio range. When the distance between two nodes are greater than the radio range, the intermediate nodes acting as routers are needed to relay data across the network. Due to the node mobility, the link between two nodes breaks frequently, which finally influence the system performance in terms of the throughput and the average transmission delay. Using stable links is crucial to obtain good performance in MANETs. Therefore, it is useful to know the link stability, such as the probability of link availability, link holding time, or

link breaking probability, before designing a MANET. In recent years, some research work on the links of MANETs has been done either by means of simulation [1], [2] or by means of analytical models [3], [4]. Most of the analytical models focus themselves on the link availability evaluation or prediction. In [3], an analytical model is presented for the probability of link availability at time  $t_0 + t$ , given that a link is available at time  $t_0$ . In [4], it provides an analytical model to obtain the probability function for a statistical forecast of link expiration times in a two-hop scenario with two stationary nodes as source and destination and one node in motion as relay. However, these analytical models only estimate the availability of a radio link at a certain point in time. They do not take into account other factors, such as radio range, packet size, et al., which may impact the link status. In this paper, we derive an analytical model to evaluate the link status such as the average link available time, the link holding time, and the link breaking probability with a random walk-based mobility model. The knowledge of link status can serve as important groundwork for further analysis of network throughput.

Since real movement patterns are difficult to obtain, a common approach is to use synthetic mobility models which mimic the behavior of real mobile entities [5], [6], [7]. The most widely used mobility model is probably the random waypoint model. In this model, a mobile node chooses uniformly at random a destination in the simulation area and travels toward the destination with a speed chosen uniformly in the interval  $[0, V_{max}]$ . Once the mobile node reaches the destination, it will pause for a specified time period according to some random variable before starting the process again. Recently several studies of this model have revealed various flaws or unexpected properties. In [8], it has been shown that the average speed of mobile nodes decays with time. This is due to the fact that low speed nodes spend more time traveling to their destination than high speed nodes. In [9], it has been shown that the random waypoint model does not lead to a uniform distribution of node location. In this paper, we will define two random walk-based mobility models that will not suffer from these problems.

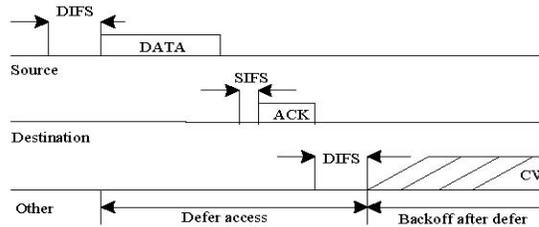
The IEEE 802.11 has been standardized [16] and been widely used by researchers for MANETs in the past few years. The IEEE 802.11 standard provides both Medium Access Control (MAC) layer and physical (PHY) layer specifications. In the standard, it defines two fundamental MAC methods: the contention-based distributed coordination function (DCF) and contention-free based point coordination function (PCF). The IEEE 802.11 DCF protocol is the most popular MAC protocol used in both wireless LANs and MANETs. The performance analysis of the IEEE 802.11 DCF has been covered in several research work [10], [12], [14], [15]. However, these models only consider the static environment in which the nodes are stationary. In this paper, the throughput of basic CSMA/CA (carrier sense multiple access with collision avoidance) and CSMA/CA with RTS/CTS (request-to-send/clear-to-send) protocols in MANETs are evaluated.

The main contribution of this paper is to provide a new framework for analyzing random walk-based models and show how the framework can be used for evaluating the link status of MANETs considering the node's mobility. The throughput of basic CSMA/CA and CSMA/CA with RTS/CTS protocols with mobility in single-hop and multi-hop MANETs are also evaluated.

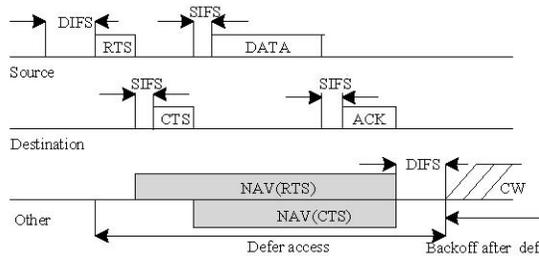
The rest of this paper is organized as follows. In the next section we briefly describe the CSMA/CA protocol. The link characteristic model is presented in Section III. Section IV gives the impact of mobility on the throughput of CSMA/CA protocol. Some numerical results and discussions are provided in Section V. Finally, we conclude the paper in Section VI.

## II. DCF PROTOCOL

This section briefly summarizes the IEEE 802.11 distributed coordination function (DCF) as it is specified by the IEEE 802.11 MAC protocol. For a more complete specification, please refer to [16]. The IEEE 802.11 standard defines the fundamental MAC protocol known as CSMA/CA. In the IEEE 802.11, there are two access protocols, namely, basic CSMA/CA and CSMA/CA with RTS/CTS. With the basic CSMA/CA protocol (see Fig. 1), a node, before initiating a transmission, senses the medium to determine if any other node is transmitting. The node proceeds with its transmission if the medium is idle for an interval that exceeds the distributed interframe space (DIFS). If the medium is busy, the node will defer transmission until the end of the current transmission. Prior to transmission again, the node will initiate a backoff interval, a random interval being selected from  $[0, CW]$  ( $CW$  is the contention window) to initiate the backoff timer. The backoff timer is decremented only when the medium is idle, and it is frozen when the medium becomes busy. After a busy period, the backoff time resumes only after the medium is idle longer than DIFS. A node initiates a transmission when the backoff timer reaches zero. If the packet is successfully received at destination, the receiver will send an acknowledgement (ACK) back to the sender after a short interframe space (SIFS). In CSMA/CA with RTS/CTS (see Fig. 2), whenever a node wishes to send a data packet, it will broadcast a short RTS containing the length of the data frame that will follow. Upon receiving the RTS, the destination responds by broadcasting a CTS packet which also contains the length of the upcoming data frame. Any node hearing either of these two control packets must be silent long enough for the data packet to be transmitted. After this exchange, the transmitter will begin the packet transmission.



**Fig. 1 Basic CSMA/CA potocol**



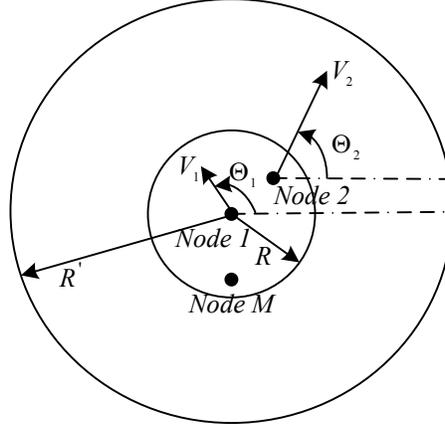
**Fig. 2 CSMA/CA with RTS/CTS protocol**

## III. LINK CHARACTERISTIC MODEL

### 3.1. Mobility Model

In our research, the system consists of  $M$  nodes and the nodes are distributed over a circular area with radius  $R'$  according to a two-dimensional uniform distribution at the beginning. At later their positions are changed according to the mobility models described below. In this paper, we assume that all nodes have the same radio range  $R$ . Two random walk-based mobility models are defined in this paper. Mobility model I is described as follows: at the beginning, node  $i$  ( $i = 1, 2, \dots, M$ ) chooses

its moving speed  $V_i$  (random variable) with a probability density function (pdf)  $f_v(v)$  and moving direction  $\Theta_i$  (random variable) with a pdf  $f_\theta(\theta)$ . The mobile nodes will keep their moving speeds and directions. In mobility model II, node  $i$  picks a direction  $\Theta_i$  at random with a pdf  $f_\theta(\theta)$  and moves in that direction for a time period  $T_i$  (random variable) with a pdf  $f_T(t)$ , at speed  $V_i$ , where the moving speed  $V_i$  is a random variable with a pdf  $f_v(v)$ . After node  $i$  spends time  $T_i$  in  $\Theta_i$  direction, it chooses a new moving direction and moving speed. The node then continues along this path for a new time period. The node distribution and mobility models are shown if Fig. 3.



**Fig. 3 Mobility model**

### 3.2. Analysis of Node Mobility

Unlike the cellular system where the base station is fixed, all nodes in MANETs are mobile. There exists relative mobility among nodes, which increases the difficulty to analyze the system. In this paper we use a transform method by considering one node to be stationary while the other node moves with a relative velocity to the first one.

We assume that node 1 moves with a velocity  $\vec{V}_1$  and node 2 moves with a velocity  $\vec{V}_2$ . The relative velocity  $\vec{V}$  of node 2 to node 1 is given by

$$\vec{V} = \vec{V}_2 - \vec{V}_1 \tag{1}$$

The magnitude  $V$  of  $\vec{V}$  is given by

$$V = \sqrt{V_1^2 - 2V_1V_2 \cos(\theta_1 - \theta_2) + V_2^2} \tag{2}$$

where  $V_1$  and  $V_2$  are magnitudes of  $\vec{V}_1$  and  $\vec{V}_2$ .

The mean value of  $V$  is given by

$$E(V) = \int_{V_{\min}}^{V_{\max}} \int_{V_{\min}}^{V_{\max}} \int_0^{2\pi} \int_0^{2\pi} \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \cos(\theta_1 - \theta_2)} \cdot f_{V_1, V_2, \Theta_1, \Theta_2}(v_1, v_2, \theta_1, \theta_2) d\theta_1 d\theta_2 dv_1 dv_2 \tag{3}$$

where  $f_{V_1, V_2, \Theta_1, \Theta_2}(v_1, v_2, \theta_1, \theta_2)$  is the joint pdf of the random variables  $V_1$ ,  $V_2$ ,  $\Theta_1$ , and  $\Theta_2$ ,  $V_{\min}$  and  $V_{\max}$  are the minimum and maximum moving speeds, the symbol  $E[X]$  is used to represent the average value of a random variable  $X$  throughout the paper. Since the moving speeds  $V_1$  and  $V_2$  and moving directions  $\Theta_1$ , and  $\Theta_2$  of nodes 1 and 2 are independent, equation (3) can be simplified as

$$E(V) = \int_{V_{\min}}^{V_{\max}} \int_{V_{\min}}^{V_{\max}} \int_0^{2\pi} \int_0^{2\pi} \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \cos(\theta_1 - \theta_2)} \cdot f_V(v_1)f_V(v_2)f_{\Theta}(\theta_1)f_{\Theta}(\theta_2)d\theta_1d\theta_2dv_1dv_2. \quad (4)$$

If  $\Theta_1$ , and  $\Theta_2$  are uniformly distributed in  $(0, 2\pi]$ , equation (4) can be further rewritten as

$$E(V) = \frac{1}{\pi^2} \int_{V_{\min}}^{V_{\max}} \int_{V_{\min}}^{V_{\max}} \int_0^{2\pi} \int_0^{2\pi} (v_1 + v_2) \cdot F_e\left(\frac{2\sqrt{v_1v_2}}{v_1 + v_2}\right) f_V(v_1)f_V(v_2)dv_1dv_2 \quad (5)$$

where  $F_e(k) = \int_0^1 \sqrt{\frac{1-k^2t^2}{1-t^2}}$  is complete elliptic integral of the second kind.

Therefore, in the following analysis, we can consider that node 1 is at rest, and node 2 is moving at a relative velocity instead of the two nodes moving with their respective velocities.

### 3.3. Radio Range Dwell Time of a Mobile Node

The radio range dwell time  $T_{dwell}$  (random variable) of a mobile node is defined as the time spent within its link partner's radio range after a packet is generated from this node. Generally, the radio range dwell time is related to the mobility model. One simple mobility model used in the cellular system is the fluid flow model, which assumes a uniform density of mobile nodes throughout the area and nodes are equally likely to move in any direction. In MANETs, under mobility equilibrium condition, the system with model I or II does not favor any specific location. Hence, both models I and II distribute mobile nodes uniformly everywhere. Using the transform method, [3] shows that the direction of  $\vec{V}$  is distributed uniformly over  $[0, 2\pi)$ . Therefore, we can consider that the mobility models described in this paper are also fluid flow model. For a two-dimensional fluid flow model, the mean rate of crossing the boundary of its link partner's radio range [11] is given by

$$\mu_{dwell} = \frac{E[V]L}{\pi A}, \quad (6)$$

where  $A$  is the area of the radio range and  $L$  is the perimeter of this area. Therefore, the average radio range dwell time of a mobile node is given by

$$E[T_{dwell}] = \frac{\pi A}{E[V]L} \quad (7)$$

### 3.4. Link Holding Time

The time period from a packet generation to its successful transmission is defined as a virtual packet transmission time  $T_v$  (random variable) in this paper. For MANETs with DCF protocol, the virtual packet transmission time includes the defer time, backoff time, packet transmission time, propagation delay, and so on. When the node is mobile, the packet transmission will fail if the node moves out of its link partner's radio range before the successful packet transmission. Therefore, the link holding time  $T_{lh}$  (random variable) is equal to the smaller of the radio range dwell time  $T_{dwell}$  and the virtual packet transmission time  $T_v$ . We have

$$T_{lh} = \min(T_{dwell}, T_v) \quad (8)$$

Since the radio range dwell time  $T_{dwell}$  and the virtual packet transmission time  $T_v$  are independent, we can get the pdf of the link holding time  $T_{lh}$  by

$$f_{T_{lh}}(t) = f_{T_{dwell}}(t)[1 - F_{T_v}(t)] + f_{T_v}[1 - F_{T_{dwell}}(t)], \quad (9)$$

Where  $f_{T_{dwell}}(t)$  is the pdf of the radio range dwell time,  $f_{T_v}(t)$  is the pdf of the virtual packet trans-

mission time,  $F_{T_v}(t)$  is the cumulative distribution function (CDF) of the virtual packet transmission time, and  $F_{T_{dwell}}(t)$  is the CDF of the radio range dwell time.

### 3.5. Link Breaking Probability

When a node generates a packet, subsequent crossing the boundary of its link partner's radio range while the virtual packet transmission is still in progress makes the link break. The link breaking probability  $P_{lb}$  of a communication pair is the probability that the virtual packet transmission time exceeds the radio. Thus, we have

$$P_{lb} = P(T_{dwell} < T_v) \quad (10)$$

Since  $T_v$  and  $T_{dwell}$  are independent, we have

$$P_{lb} = \int_0^{\infty} \int_0^t f_{T_v}(t) f_{T_{dwell}}(u) du dt \quad (11)$$

Previous research into the teletraffic of mobile communications has used the cell dwell time with a generalized gamma distribution [17], a hyperErlang distribution [18], and so on. For the sake of simplicity, many researchers have assumed, either explicitly or implicitly, the cell dwell time to be an exponentially distributed random variable [11]. In a special case, we assume that the radio range dwell time and the virtual packet transmission time have negative exponential distributions with mean rates  $\mu_{dwell}$  and  $\mu_v$ , respectively. Therefore, we have

$$\mu_{dwell} = \frac{\mu_{dwell}}{\mu_{dwell} + \mu_v}. \quad (12)$$

Substituting equation (6) into (12), Equation (12) can be rewritten as follows

$$P_{lb} = \frac{E[V]L}{\mu_v \pi A + E[V]L}. \quad (13)$$

The concept of link breaking probability can be extended to an entire routing path. The entire path will break if at least one of the links breaks during the communication period. Since any two links without sharing node i.i.d and any two links with one common node cannot transmit the packets at the same time, we can consider that each link fails independently. Therefore, the path breaking probability is given by

$$P_{pb} = 1 - (1 - P_{lb})^n, \quad (14)$$

where n is number of links (hops) of the path.

## IV. PERFORMANCE ANALYSIS

The other main contribution of this paper is the analytical model for the throughput of CSMA/CA protocol in presence of mobility. In the analysis, we assume that the packets have fixed size and the packet transmission time is  $T_p$ . The time is slotted and a node is allowed to transmit only at the beginning of each slot time. The time is normalized by  $T_p$ . That is, the packet transmission time is equal to 1. The slot time size  $\alpha$  is set to propagation delay. Nodes can be in one of the two states: thinking or backlogged. In the thinking state, a node generates a new packet in a time slot with probability g, and a node is said to be backlogged state if its packet either had a channel collision or has been blocked because of a busy channel. A node can switch from backlogged state to thinking

state if it completes a successful transmission and switch back to backlogged state if it has a collision or blocked because of busy channel. If we assume that the total traffic load  $G$  which is the number of total packets during one normalized unit time is Poisson process, we have  $Mg = G\alpha$  [20].

#### 4.1. Performance of single-hop MANETs

In a single-hop MANETs, all the nodes can hear each other, which means it is a fully connected network. Therefore, we can set  $R' = \frac{R}{2}$ . To maintain the traffic load, we assume that the number of nodes in the single-hop network is fixed. The node chooses its destination uniformly in its radio range. Since there are  $M$  nodes in the system, the number  $N^t$  of backlogged nodes at the beginning of slot time  $t$  can be  $0, 1, 2, \dots, \text{ or } M$ . For simplicity, we assume that each backlogged node has the same probability  $\gamma_i$  to transmit the packet when  $N^t$  equals to  $i$ . Therefore, we can obtain the probability that the channel is idle at slot time  $t$  is equal to  $\delta_i = (1 - \gamma_i)^i (1 - g)^{M-i}$ . The packet transmission will succeed if only one node is transmitting. Thus, the probability of successful packet transmission is given by

$$P_{si} = \frac{(M - i)g(1 - g)^{M-i-1}(1 - \gamma_i)^i + i\gamma_i(1 - \gamma_i)^{i-1}(1 - g)^{M-i}}{1 - (1 - \gamma_i)^i(1 - g)^{M-i}}. \quad (15)$$

In the above equation, the probability  $\gamma_i$  is yet to be determined. In [14], an analytical model is developed to compute the collision probability  $P_{ci}$ , which is given as

$$P_{ci} \frac{1 - P_{ci} - 2^m P_{ci}^{m+1}}{1 - 2P_{ci}} = \frac{2}{W} \left(1 + \frac{2i}{3}\right) \frac{i-1}{i}, \quad i > 1, \quad (16)$$

where  $W$  is the minimum contention window and  $m$  is to determine the maximum contention window  $CW_{\max}$  and it satisfies  $CW_{\max} = 2^m W$ . Note that the collision probability  $P_{ci}$  is the probability that more than one node transmit at the same slot. This yields to

$$P_{ci} = 1 - (1 - \gamma_i)^{i-1}(1 - g)^{M-i} - (1 - \gamma_i)^i \cdot (1 - g)^{M-i-1} - (1 - \gamma_i)^i(1 - g)^{M-i}. \quad (17)$$

Then we can get the probability  $\gamma_i$  ( $i > 1$ ) from Equations (16) and (17). Obviously,  $\gamma_0 = 0$ , and

$$\gamma_1 = \frac{1}{W}.$$

From Fig. 1, we know that the length of successful transmission time for the basic CSMA/CA includes the packet transmission time  $T_p$  plus a propagation delay  $\alpha$ , a SIFS delay  $T_{SIFS}$ , transmission time of ACK message  $T_{ACK}$  plus a propagation delay, and a DIFS delay  $T_{DIFS}$ . That is

$$T = T_{DIFS} + T_p + T_{SIFS} + T_{ACK} + 2\alpha. \quad (18)$$

Here we add a propagation delay to each transmission since the channel will be sensed idle only after a propagation delay once a packet is transmitted. In the basic CSMA/CA, the collision occurs in the packet transmission. When a collision occurs, the sender will not receive the ACK message after a SIFS delay. Thus the length of collision is given by

$$C = T_{DIFS} + T_p + T_{SIFS} + \alpha. \quad (19)$$

The idle period is geometrically distributed. Its expectation is given by

$$\bar{T}_i = \frac{1}{1 - (1 - \gamma_i)^i(1 - g)^{M-i}}. \quad (20)$$

In [10], we have obtained the stationary probability  $\pi_i$  that  $i$  nodes are backlogged using linear feedback model. Let  $S$  be the system throughput, which is defined as the fraction of channel time occupied by a valid transmission. Therefore, the throughput of basic CSMA/CA protocol is given by

$$S = \frac{\sum_{i=0}^M \pi_i P_{si} T_p}{\sum_{i=0}^M \pi_i [\bar{T}_i + 1 + P_{si} T + (1 - P_{si}) C]} \quad (21)$$

Similarly, for the CSMA/CA with RTS/CTS, the lengths of successful transmission and collision are given by

$$T = T_{DIFS} + T_{RTP} + T_{CTS} + T_p + T_{ACK} + 3T_{SIFS} + 4\alpha \quad (22)$$

and

$$C = T_{DIFS} + T_{RTS} + T_{SIFS} + \alpha. \quad (23)$$

Substituting the above  $T$  and  $C$  into equation (21), we can get the throughput for CSMA/CA with RTS/CTS.

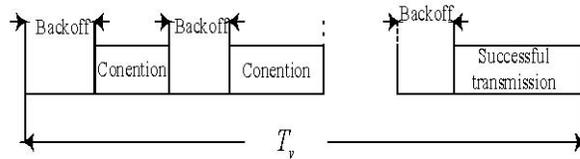
Now, we will take the node mobility into consideration. For MANETs with CSMA/CA protocols, when more than one node are active, the virtual packet transmission time includes a successful packet transmission time, backoff time, and contention time i.e. a collision or transmission by another node.

Fig. 4 shows that before a successful transmission contentions may occur along with periods in which the medium is idle due to the backoff algorithm. The mean value of the virtual packet transmission time is given by

$$E[T_v] = E\left[T_b + \sum_{i=0}^N T_i + T\right], \quad (24)$$

where  $T_b$  is the length of backoff time,  $T_i$  is the length of contention time, and  $N$  is the average number of contentions during a virtual packet transmission period. The average backoff time [14] is equal to

$$E[T_b] = \sum_{i=1}^M \pi_i \frac{1 - P_{ci} (2P_{ci})^m}{1 - 2P_{ci}} \frac{W}{2}. \quad (25)$$



**Fig. 4 Structure of time  $T_v$**

The length of contention is either equal to the length of successful packet transmission or the length of collision. From the result of [14], the average number of contentions is obtained by

$$N = \sum_{i=1}^M \frac{2}{3} \pi_i (i - 1). \quad (26)$$

In our analysis, we assume that  $N \sum_{i=0}^M P_{si} \pi_i$  contentions have the length of successful packet

transmission  $T$  and the rest contentions have the length of collision  $C$ . If we assume that the virtual packet transmission time  $T_v$  is negative exponential distributions with mean  $E[T_v]$ , the link breaking probability for CSMA/CA protocols can be calculated easily by Equation (13).

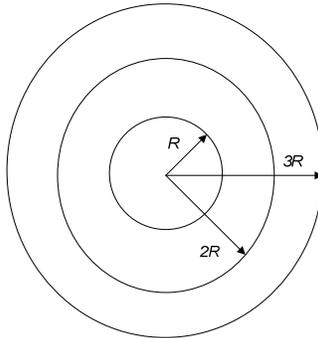
Therefore, when considering the node mobility, the system throughput  $S'$  is equal to

$$S' = \frac{\sum_{i=0}^M \pi_i (1 - P_{lb}) P_{si} T_p}{\sum_{i=0}^M \pi_i \{ \bar{T}_i + 1 + (1 - P_{lb}) P_{si} T + [1 - (1 - P_{lb}) P_{si}] C \}}. \quad (27)$$

The numerator is the time that the channel is used to successfully transmit the payload during a cycle and the denominator is the average time of a cycle (an idle period followed by a busy period is called a cycle).

### 4.2. Performance of multi-hop MANETs

A multi-hop ad hoc network has the advantage that multiple concurrent transmissions can take place simultaneously at geographically separated locations. On the other hand, such a capacity gain may be offset by the hidden node problem and the extra hops needed for a packet to reach its destination. The latter is greatly influenced by the routing protocol adopted. In this section, we focus on the impact of the hidden node problem. To eliminate the effect of routing, all packets generated from a node are assumed to be destined to its neighbor nodes. Fig. 5 shows the network model used in the analysis. We still assume that the radio transmission range is  $R$  and an average of  $M$  nodes is within this range. Since we cannot generate an infinite network model, we just focus our attention on the performance of the innermost  $M$  nodes. In order to make sure that all hidden nodes of the innermost  $M$  nodes are included, the range of the service area is chosen to be  $3R$ .

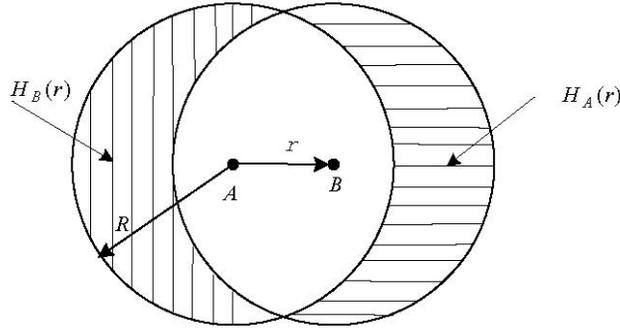


**Fig. 5 Multi-hop Network Model**

The impact of hidden terminals on the performance of wireless multi-hop MANETs depends on the number of nodes within the radio range of the sender and the receiver. We still assume that node density is the same as the single-hop MANET. Fig. 6 gives the hidden area HA of the transmitting node A and the hidden area HB of the receiving node B.  $H_A(r)$  is given by

$$H_A(r) = \pi R^2 - 2R^2 F_q \left( \frac{r}{2R} \right), \quad (28)$$

where  $F_q(t) = \arccos(t) - t\sqrt{1-t^2}$ .



**Fig. 6 Illustration of hidden area**

Given that all nodes choose their moving directions uniformly in  $(0, 2\pi]$ , [21] shows that the steady state distribution of the mobile nodes over the circular area is still uniform. If node  $A$  chooses any one of its neighbors with equal probability and the average number of nodes within a region of radius  $r$  is proportional to  $r^2$ , thus the pdf of the distance  $r$  is given by

$$f(r) = \frac{2r}{R^2}, \quad 0 \leq r \leq R. \quad (29)$$

Therefore, we can calculate the average number of hidden nodes of  $A$

$$N_h = M \frac{\int_0^R \frac{2r}{R^2} H_A(r) dr}{\pi R^2}. \quad (30)$$

For basic CSMA/CA, the hidden nodes can cause collision in transmission of both data and ACK messages. First, we consider the transmission of a data packet from node  $A$  to node  $B$ . There are two cases of hidden node problems that will make this transmission fail: at least one node in the hidden area  $H_A(r)$  transmits the data or ACK messages during this packet transmission period. Based on the results from [10], the probability of the hidden nodes to transmit the data message is equal to

$$P_1 = P_{bi} \frac{N_h}{M}, \quad (31)$$

where  $P_{bi} = \frac{\overline{B}_i}{\overline{B}_i + \overline{I}_i}$  is the probability that the channel will be sensed busy.  $\overline{B}_i$  is the average channel busy period and  $\overline{I}_i$  is the average channel idle period given  $N^t = i$ . The alternative case that at least one node within the hidden area  $H_A(r)$  transmits the ACK message with probability

$$P_2 = P_{si} P_{hri} = P_{si} \frac{N_h}{M-1}, \quad (32)$$

where  $P_{si}$  is the probability of a successful transmission and  $P_{hri} = P_{bi} \frac{N_h}{M-1}$  is the probability that at least one node in  $H_A(r)$  receives the data. This is an overestimate, since some transmissions between the nodes of the hidden area have been counted in probability  $P_1$ . A better estimate is given by

$$P_2 = P_{si} P_{hri} \left(1 - \frac{N_h}{2M}\right). \quad (33)$$

Now, we consider the transmission of ACK message when node  $B$  successfully receives the data packet from node  $A$ . According to the symmetry, we know that the number of hidden nodes of  $B$  is the same as that of  $A$ . After node  $A$  finishes its transmission, all other nodes within the radio range of  $A$  should wait  $T_{DIFS}$  period before they can sense the channel. Therefore, the exact period in which the hidden nodes of  $B$  can cause collision is  $T_{ACK} + T_{SIFS} - T_{DIFS}$ . The average number of backlogged hidden nodes for  $B$  is equal to  $M_{hbi} = \frac{i}{M} N_h$ . Note that a node can start a transmission only when the channel is idle. Let  $T_{ACKi}$  be

$$T_{ACKi} = \frac{\bar{I}_i}{I_i + B_i} (T_{SIFS} + T_{ACK} - T_{DIFS}). \quad (34)$$

Therefore, the probability that at least one hidden node in  $H_B(r)$  transmits a data packet during the period  $T_{ACKi}$  is given by

$$P_3 = 1 - (1 - \gamma_i)^{T_{ACKi} M_{hbi}} (1 - g)^{T_{ACKi} M_{hbi}}. \quad (35)$$

Because the probability that the hidden nodes of  $B$  respond ACK during the period  $T_{ACKi}$  is small, we ignore it.

Based on the above discussion, the probability that node  $A$  successfully transmits the data packet to node  $B$  totally degrades  $P_d = P_1 + P_2 + (1 - P_1 - P_2)P_3$ .

For CSMA/CA with RTS/CTS, although the RTS/CTS packet exchange reduces the hidden node problem, the problem is not completely resolved. We still assume that at a time, node  $A$  is transmitting a data packet to node  $B$ , and  $i$  nodes within the radio range of node  $A$  are backlogged. For CSMA/CA with RTS/CTS, if any of the hidden nodes of  $A$  is transmitting RTS or CTS messages when node  $A$  is transmitting a RTS, the transmission of node  $A$  will fail. The corresponding  $P_1$  and  $P_2$  can be derived like basic CSMA/CA.

The collision may also happen during the data packet transmission period from node  $A$  to node  $B$ . This is because some of hidden nodes of node  $A$  will not successfully receive the CTS message and these nodes may transmit the RTS or CTS messages during this data packet transmission period. The probability that the hidden nodes will not successfully receive the CTS message is the same as the probability ( $P_{hri}$ ) that at least one of the hidden nodes of node  $A$  is receiving. Equation (39) gives the probability that at least one of the hidden nodes of node  $A$  transmit RTS, and Equation (41) gives the probability that at least one of the hidden nodes of node  $A$  transmits the CTS. Therefore, the probability that the transmission of hidden nodes of node  $A$  collide with the transmission from node  $A$  to node  $B$  is given by

$$P_3 = P_{hri} (P_1 + P_2). \quad (36)$$

The hidden nodes in hidden area  $H_B(r)$  can also cause collision to the CTS and ACK messages. In fact, compared with the probability that the RTS or data packet collides, the probability of CTS or ACK collision is very small and we can ignore them.

Based on the above discussion, the probability that the node  $A$  successfully transmits a data packet to node  $B$  is totally degrades  $P_d = P_1 + P_2 + (1 - P_1 - P_2)P_3$ .

Let  $S$  be the throughput of the entire innermost  $M$  nodes. Based on a single station standpoint, the probability of successful data packet transmission in presence of mobile and hidden nodes is equal to

$$P'_{si} = P_{si} (1 - P_{lb}) (1 - P_d). \quad (37)$$

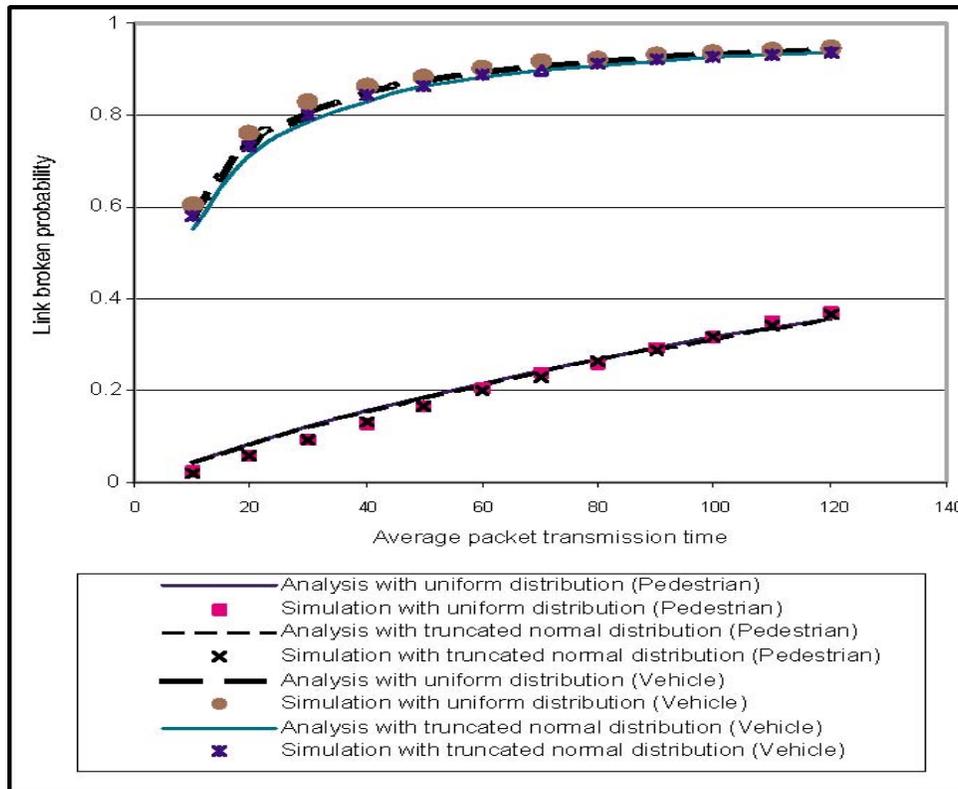
The throughput  $S$  is given by

$$S = \frac{\sum_{i=0}^M \pi_i P'_{si} T_p}{\sum_{i=0}^M \pi_i \left[ \bar{I}_i + 1 + P'_{si} T + (1 - P'_{si}) C \right]}. \quad (38)$$

## V. NUMERICAL RESULTS AND DISCUSSIONS

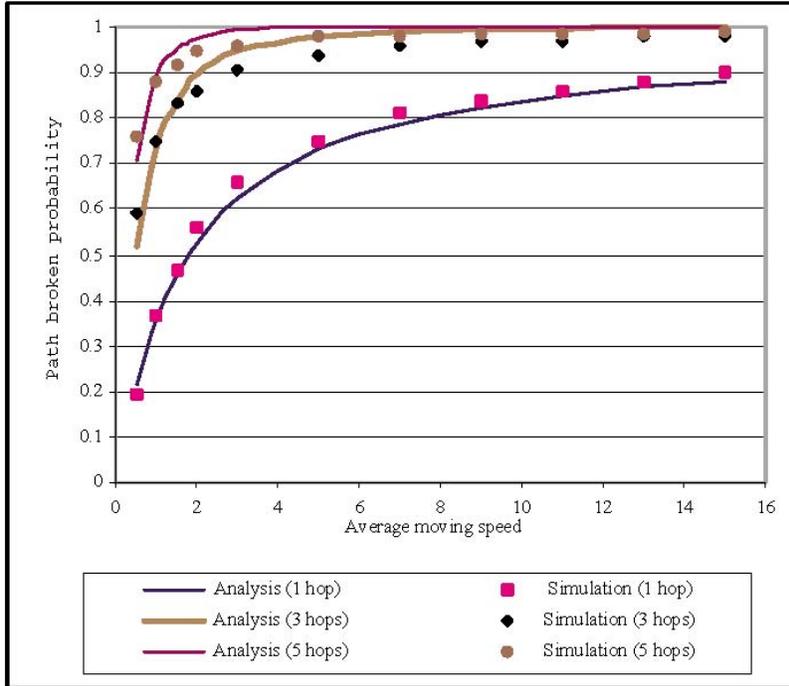
Numerical values are computed in this section. Two distributions of moving speeds are simulated: one is uniform distribution and the other is truncated normal distribution with mean  $\mu$  and standard variance  $\sigma$ . Two types of mobile users are considered, i.e., pedestrian and vehicle. The pedestrian is with following parameters: for uniform distribution,  $V_{\min} = 0m/s$ ,  $V_{\max} = 1m/s$ ; for truncated normal distribution,  $V_{\min} = 0m/s$ ,  $V_{\max} = 1m/s$ ,  $\mu = 0.5$  m/s, and  $\sigma = 0.27$  m/s; The vehicle is with following parameters: for uniform distribution,  $V_{\min} = 10m/s$ ,  $V_{\max} = 20m/s$ ; for truncated normal distribution,  $V_{\min} = 10m/s$ ,  $V_{\max} = 20m/s$ ,  $\mu = 15$  m/s, and  $\sigma = 1.91$  m/s. The moving directions are distributed uniformly in  $[0, 2\pi)$ . Time period  $T$  is exponential distribution with mean  $E[T] = 60$  s. The radio range is chosen as  $R = 100$  m. The distribution of the virtual packet transmission time is assumed to be exponential distribution.

Fig. 7 shows the impact of virtual packet transmission time on the link breaking probability for two different types of mobile users. We observe that for a given average node moving speed, the link breaking probability increases with increasing average virtual packet transmission time. Observe that for a fixed virtual packet transmission time, with increasing node moving speed the link breaking probability improves since the node with higher speed moves quickly apart from its link partner.



**Fig. 7 Link breaking probability vs virtual packet transmission time in mobility model I**

Fig. 8 shows the path breaking probability versus average moving speeds for path lengths of 1, 3, and 5 hops, respectively. Since the link and path breaking probabilities are only determined by the average moving speed if the radio range and virtual packet transmission time are given, we simulated only uniform distribution with average from 0.5 m/s to 15 m/s. For given average virtual packet transmission time, the path breaking probability increases with increasing hops. The path with more than one link (multi-hop) demonstrates convergence characteristics similar to those of the link (single-hop) breaking probability case. The radio range and the moving speed have the opposite influence on the link or path breaking probability, since scaling the radio range up has the same influence of scaling the moving speed down.



**Fig. 8 Path breaking probability vs average moving speed in mobility model I**

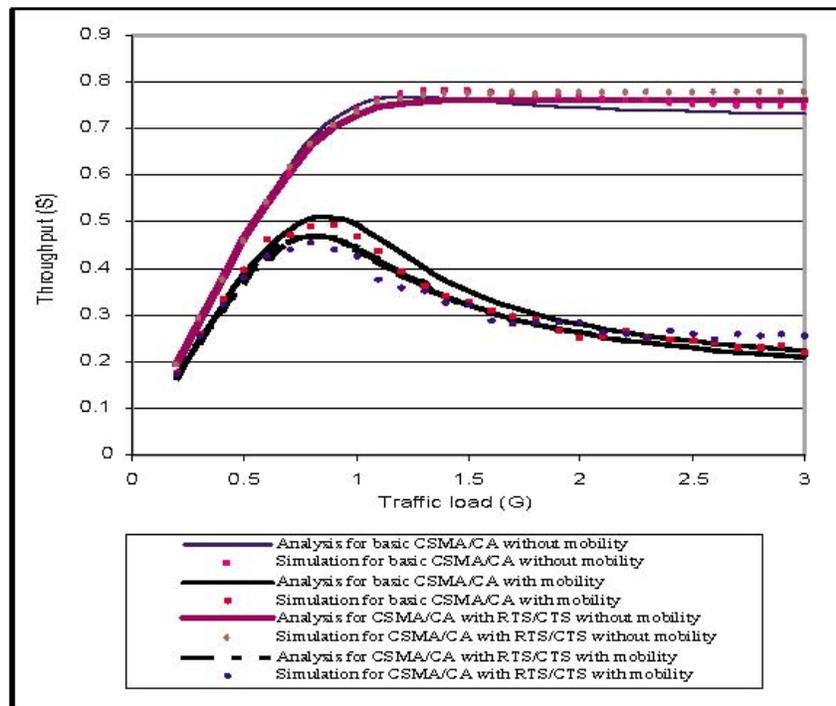
The results for mobility model II have the similar results as mobility model I, which are not included in this paper.

The simulation parameters used for throughput in single-hop and multi-hop MANETs are defined as follows:  $T_p = 1$  (i.e., 100 slots),  $\alpha = 0.01$ ,  $T_{DIFS} = 0.05$ ,  $T_{SIFS} = 0.01$ ,  $T_{RTS} = 0.05$ ,  $T_{CTS} = 0.05$ ,  $T_{ACK} = 0.05$ ,  $W = 32$ ,  $CW_{max} = 1024$ ,  $M = 15$ , and  $R = 100$  m. In the simulation, the moving speed is uniformly distributed in  $[0.01, 0.02]$ . The moving direction is distributed uniformly in  $[0, 2\pi]$ .

Fig. 9 shows the analytical and simulation results for the basic CSMA/CA and CSMA/CA with RTS/CTS protocols. From the results, it can be seen that the throughput with mobility decreases compared with the throughput without mobility. This is because the node's mobility makes some nodes move out of the radio range during the virtual packet transmission period. We can see that the results obtained from the analytical model are observed to match well with the simulation results.

## VI. CONCLUSIONS

In this paper, a theoretical model has been established to analyze the performance of wireless MANETs with mobile nodes. The model is useful for analyzing the performance parameters such as the average radio range dwell time of a mobile node, link holding time, link and path broken probabilities, throughputs of basic CSMA/CA and CSMA/CA with RTS/CTS protocols. The model is capable of accommodating many other parameters of interest as well. The results showed that the overall performance degrades with mobility as compared to those without mobility. The model has been analyzed under a general traffic load with a general moving speed and direction.



**Fig. 9 Throughput of CSMA/CA protocols**

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