A Graphical Teaching Aid to Illustrate Fourier Harmonics

by

Gene L. Harding
glhardin@purdue.edu
Elec. & Comp. Engineering Technology
Purdue University

Christopher A. Long
calong@purdue.edu
ECET Student
Purdue University

Abstract: For many students, the subject of Fourier series harmonics is a new application of the principle of superposition and their first foray into the "frequency domain" view of electronic signals. It can be a difficult journey for some, as they try to comprehend how sine waves can be added together to create waveforms like square and triangle waves. Likewise, the conceptual link between time and frequency domains can be hard to imagine. Good visualization tools can assist the learning process and improve understanding of the math used to define the Fourier series. This project came about because the authors could not find a teaching aid to illustrate either concept. The first goal was to address the problem by creating a pair of animated graphical tools to illustrate the basic ideas and facilitate student understanding. The second goal was to share these tools with others who might find them useful.

I. Introduction

In the Electrical and Computer Engineering Technology (ECET) department of Purdue University's College of Technology, the students' first year is spent focusing on time-domain issues. This includes two courses in analog electronics, two courses in digital electronics, and a projects course. The frequency domain is introduced in an AC electronics course, ECET207, during the first semester of the sophomore year by covering filters, resonance, and Fourier series.

The basic idea of the Fourier series is that any periodic waveform is actually the sum of a DC signal and many AC signals that have been adjusted in magnitude, frequency, and phase. Moreover, the frequency of each AC signal is an integer multiple, or harmonic, of the fundamental signal's frequency. The significance of this principle is that it is possible to determine how a circuit responds to *any* periodic waveform by using simple DC and phasor AC calculations [1]. In other words, the analysis *technique* does not change for different waveforms, as long as they are periodic.

Through four years of teaching ECET207, the course instructor at Purdue's South Bend location observed many students struggling with the basic concept of superposition and the relationship between time and frequency domains. Lectures covered the principles of superposition and

Fourier series, followed by associated homework assignments, computer simulations, and laboratory experiments. What was missing, however, was an easy and intuitive aid to illustrate the basic *concepts*. The professor attempted to explain these concepts using a combination of chalkboard drawings and hand-waving, but believed a better way of communicating the ideas would be helpful.

It seems intuitively obvious that a clear and simple three-dimensional (3-D) graphical depiction of a Fourier series would be a helpful addition to ECET207 classroom instruction. Moreover, Gardner's theory of multiple intelligences [2] supports this idea, where he posits interactions between spatial intelligence and logical-mathematical intelligence that benefit comprehension in the field of engineering. The best graphic, according to Tufte [3], would be a simple one that illustrates the complexity of the Fourier series. A search to find such a visualization tool yielded some other related approaches, but nothing to graphically illustrate the aforementioned concepts.

Published literature proposes several techniques and tools for teaching Fourier, including finite-dimensional vector space framework [4] and animation to illustrate accuracy vs. number of terms for a truncated Fourier series [5]. There are also myriad applications, such as using the Fourier transform for image [6] and audio [7] processing. Nevertheless, no tools were found directly targeted at illustrating the concepts of harmonic superposition and time-vs.-frequency-domain relationship.

II. Project Description

The primary goal of this project was to create a pair of animated graphics to help illustrate Fourier series. The first graphic illustrates how a waveform, such as a square wave, can be created by the superposition of individual sine wave harmonics adjusted in amplitude and phase. To illustrate the superposition dynamically, the MATLAB display is split into two plot sections: the upper plot shows the sum of the Fourier series it develops, and the lower plot shows the superimposed sinusoidal harmonics used to create it. The animation begins with the fundamental, viewed in the time domain (Figure 1), and adds several harmonics to show how a square wave begins to take shape (Figure 2). The animation ends with the resultant square wave shown in the upper plot and several dozen of its comprising harmonics shown superimposed in the lower plot (Figure 3). This should not be too difficult a stretch for most ECET207 students, since they are presumably comfortable by this time with the concept of superposition as it relates to a periodic waveform "riding" on a DC offset.

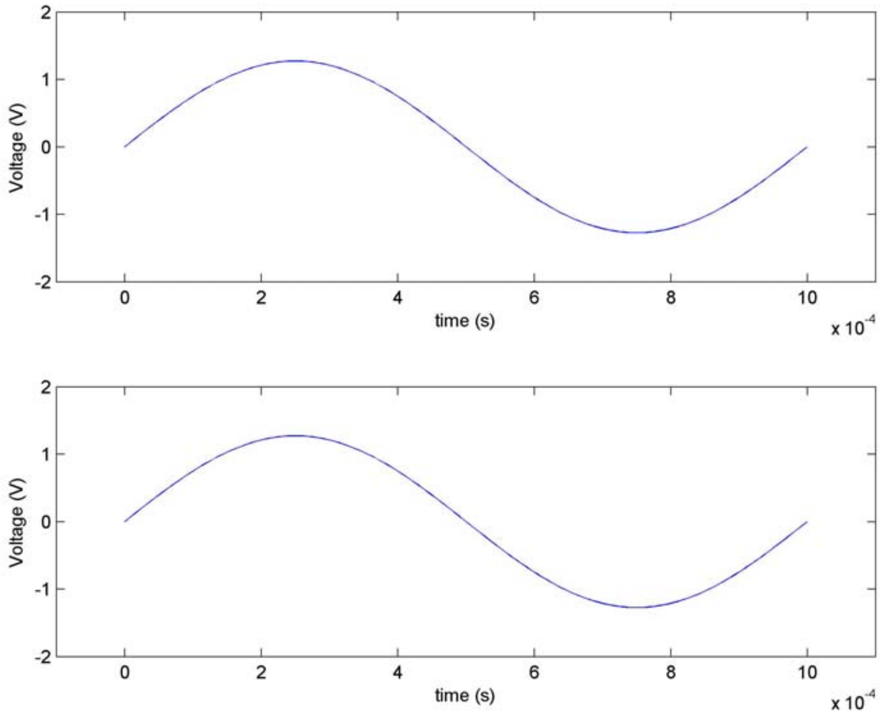

Figure 1: Fourier series (top) with fundamental only (bottom)

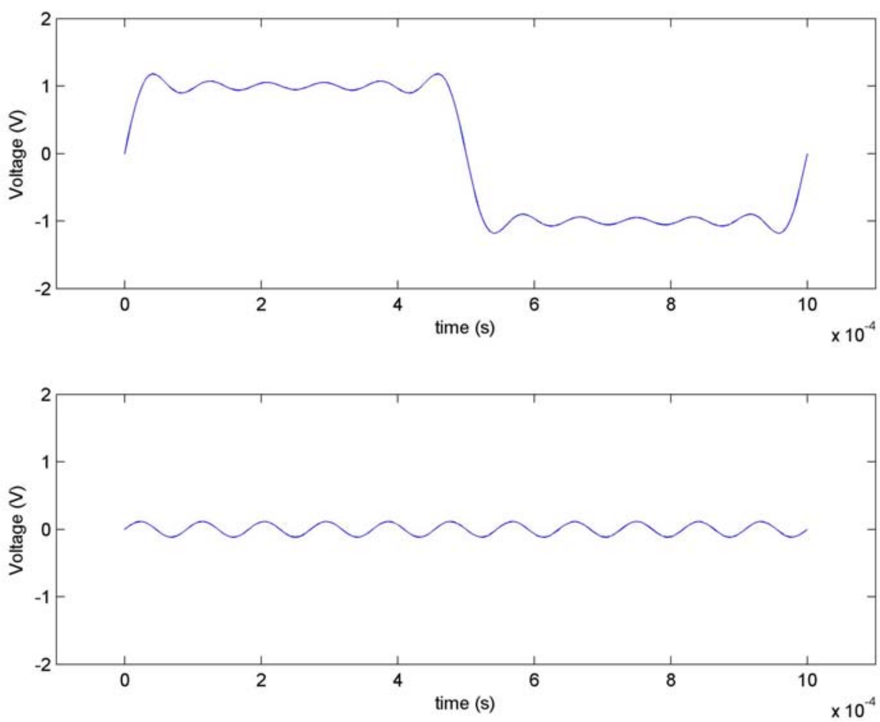

Figure 2: Fourier series sum of several harmonics (top) and the last harmonic added (bottom)

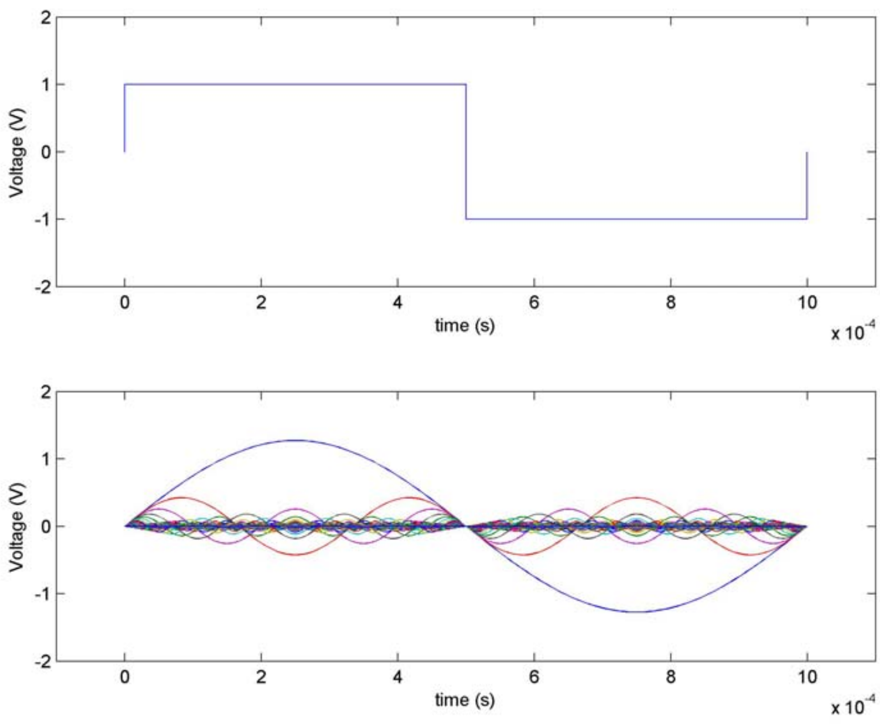

Figure 3: Resultant square wave (top) and its component harmonics (bottom)

The second graphic is intended to show the relationship between the time and frequency domains. It begins with a frequency-domain view of several harmonics of a 1-kHz square wave, each of which is a single spike on the plot, as shown in Figure 4. The animation rotates the figure 90° to the time domain. As the rotation begins, it becomes apparent that each of the spikes in the frequency domain is actually an edge view of a sine wave for the corresponding harmonic, as shown in Figure 5. At the end of the rotation the fundamental and its harmonics are viewed in an overlay fashion in the time domain (Figure 6), just before being replaced by the square wave (Figure 7). The square wave depicts the result of the entire Fourier series, not just the fundamental and five harmonics shown in the initial graphic.

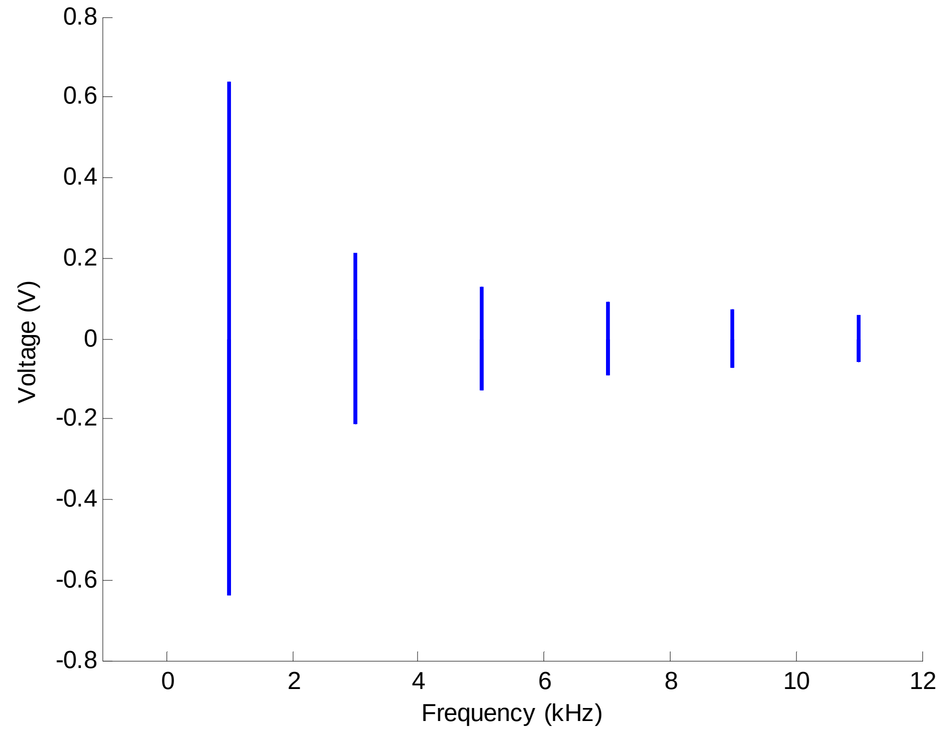

Figure 4: Fundamental and first five harmonics of a 1-kHz square wave

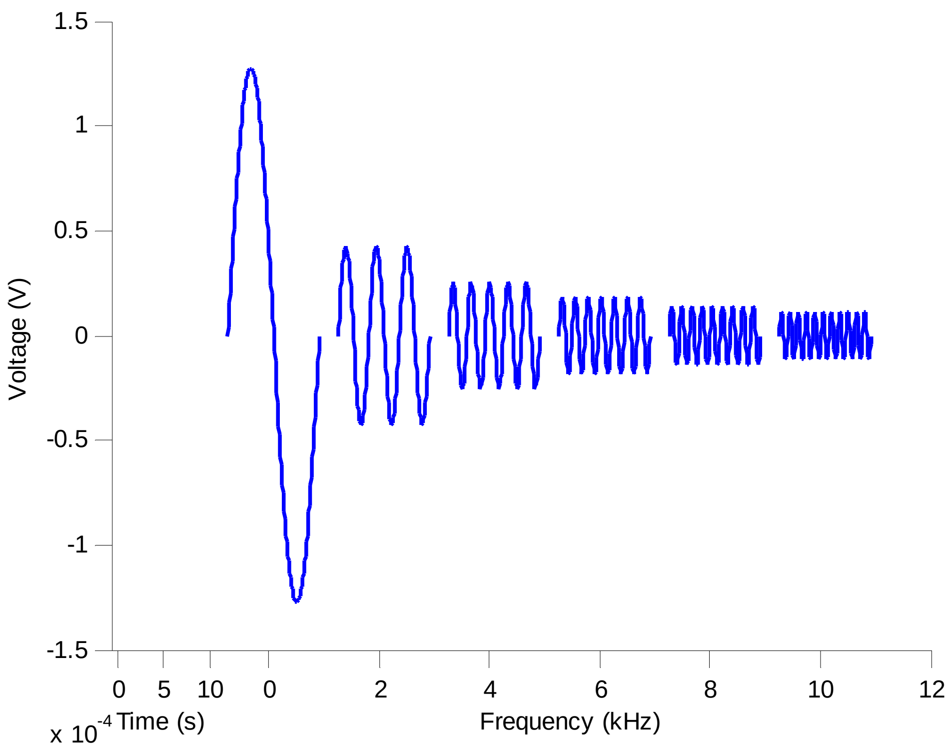

Figure 5: Fourier series rotating from frequency to time domain

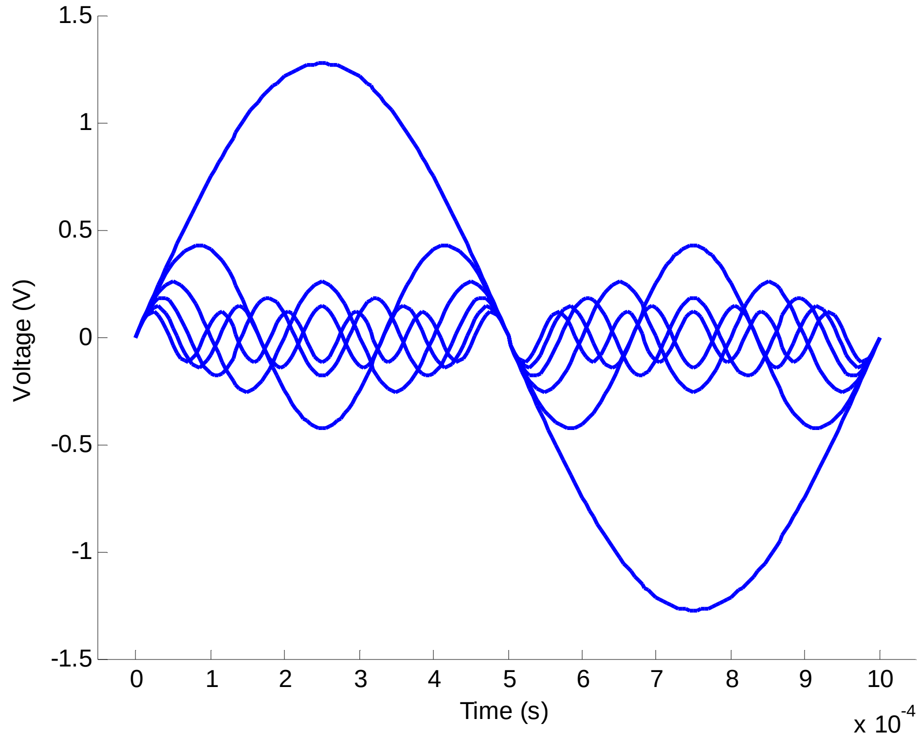

Figure 6: Fourier series fundamental and five harmonics viewed in the time domain

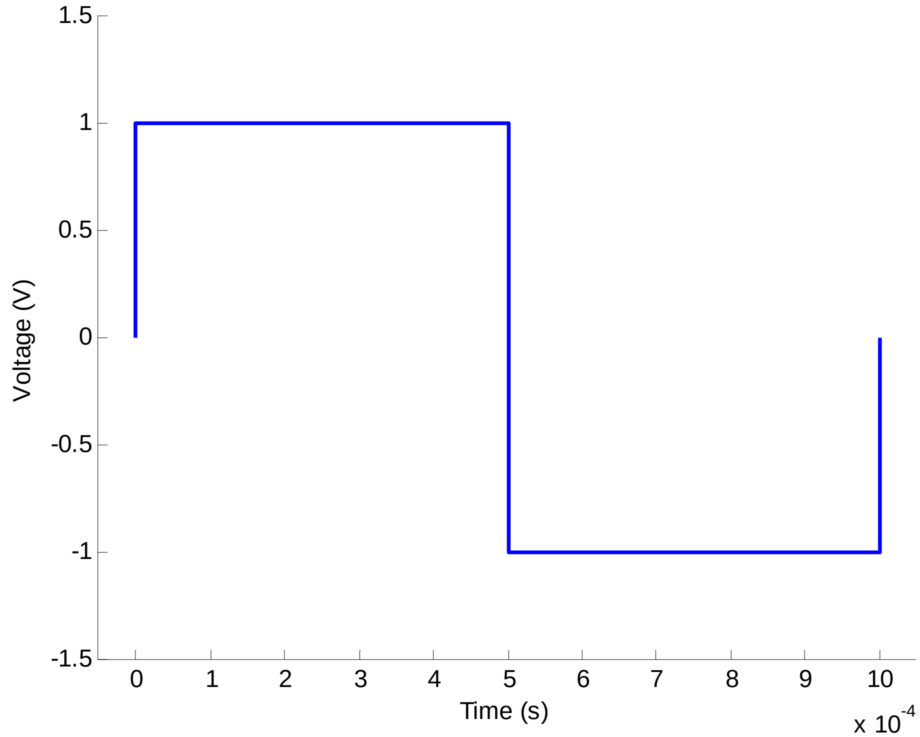

Figure 7: Square wave result of complete Fourier series

These two animated graphics provide a visual foundation for the mathematical principles of the Fourier series. They will be used as part of the classroom instruction in ECET207 for the first time during the fall 2007 semester.

A secondary goal of the project, and the reason for this article, was to make these tools available to other instructors who might find it useful for teaching Fourier series in their courses. To this end, both the animated graphics and underlying m-files are available at the following web page: http://www.tech.purdue.edu/southbend/academics/degreeprograms/ecet/fourier.cfm.

III. Animated Tool Development

The proposed graphical aids were developed by an assistant professor and third-year ECET student in the College of Technology at Purdue University. MATLAB was chosen as the development tool for the following reasons:

- It was readily available in the ECET department at Purdue;
- It has the requisite computational and graphical output capabilities;
- It is widely used as an engineering tool in both academia and industry.

The tools were developed as animated graphics using the *plot*, *plot3*, and *movie* functions within MATLAB. The plotting functions provide a direct output within MATLAB, while the movie functions enable the creation of a video output file in standard *.avi* format. For the MATLAB plot functions, the *pause* command was used to modulate the speed of the animation; for the video file, extra (i.e., redundant) frame captures were used to slow down the animation speed when desired. The Fourier series itself, in both cases, is a two-dimensional (2-D) array of values (voltages) indexed by time and harmonic number.

The basic approach taken for the superposition graphic was to split the display window into two sections then iteratively plot the harmonic sum in the upper plot and the most recent contributing harmonic in the lower plot. The video file is initially slowed down by a factor of ten by writing ten frames to the *.avi* file for each of the first several harmonics. This allows the viewer to more easily comprehend what is happening to form the harmonic sum. After this point only one frame is captured per harmonic, allowing a large number of harmonics to be added fairly quickly. After some experimentation, harmonic number 171 was chosen to be the truncation point for the Fourier series because the sum at that point so closely resembles the resultant square wave. The final plot shows the square wave that would result from the full Fourier series in the top plot and the fundamental through 171st harmonic overlaid in the bottom plot (see Figure 3). Twenty frames were captured of this final plot so the viewer would have time to clearly see it. Moreover, the delay at the end allows the user to pause the video on the final graphic. This will be useful for instructional purposes.

The time vs. frequency domain animation built upon the first tool. It used the same time index and 2-D harmonic array that the superposition tool used to plot the overlaid harmonics. However, since it required a 3-D plot, a third variable was added: a second 2-D array containing the harmonic number (i.e., frequency) corresponding to each voltage value in the harmonic array.

These three variables—time, frequency, and voltage—allowed use of the *plot3* command to create a 3-D plot of the Fourier series harmonics in both time and frequency domains. The plot is initially displayed from the perspective of the frequency domain (Figure 4), and the MATLAB *view* command is used to rotate the 3-D graphic from frequency to time domain. Extra frames were captured for the first several degrees of rotation to slow down the video by a factor of four. Once the rotation to the time domain was complete (Figure 6), twenty extra frames were taken; then the resultant square wave was plotted (Figure 7) and twenty extra frames again taken. The extra frames at the end of the video allow the viewer extra time to grasp the principle illustrated by the graphic, and make it easier to pause the animation, if needed.

IV. Conclusion

The concepts behind Fourier series can be difficult for some students to comprehend. Two of these concepts are the principle of superposition and the relationship between time and frequency domains. Good visualization tools can enhance comprehension of engineering subjects that can be represented graphically. The authors searched for tools to use for the purpose of improving quality of instruction in a 2nd-year electronics course at Purdue University, but found none. Since such tools were not readily available, the authors created them using MATLAB and have made them publicly available via Purdue University's web site (see URL in section II, above). They will be used for the first time during the fall 2007 semester in a sophomore-level electronics course.

References

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