
Measuring Young's Modulus with Metal Flatstock

by

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Abstract: A popular way to measure elastic modulus in Physics classes is to stretch a fine wire using known weights [1, 2]. Students divide the applied tensile stress by the measured tensile strain to obtain the elastic modulus. When engineering technology students take Statics and Strength of Materials courses, they learn how to apply basic physics concepts to mechanical structures, such as beams loaded in bending. This paper describes a simpler, less expensive approach for measuring elastic modulus, using the center-span deflection of a simply-supported beam. The method appeals to Strength of Materials students because it uses concepts learned in their coursework, and students respond positively to hands-on demonstrations.

I. Introduction

Most students in my Statics and Strength of Materials classes have taken high school physics, and many have completed Physics I in college, so they have a basic understanding of how materials respond to applied forces. They may have measured elastic modulus in a physics lab using a wire specimen loaded in tension. One end of the wire is wrapped around a cylindrical rod; the other end is tied to suspended deadweights. As the wire stretches elastically, its length increases in proportion to the suspended weight, and the rod rotates. Various methods are used for measuring the rotation of the rod. The equipment can be obtained off the shelf or from a scientific supply house such as Sargent-Welch [3]. In physics classes, students are taught to convert all quantities to standard units prior to solving equations, so they are not exposed to the factor-label method, which is standard in all engineering disciplines.

My Strength of Materials class consists of 25 lectures, 3 exams, and a review session before the final exam. In the past, students rarely asked enough questions to run the full length of the review session, so I looked for an inexpensive experiment that would show Mechanical Engineering Technology students that the theory they were learning actually works. I also wanted to emphasize the factor-label method of unit conversions. Since a large portion of the course deals with beams in bending, I chose an experiment that demonstrates this behavior. I purchased a steel rod, a stamped steel piece of flatstock, and an extruded aluminum piece of flatstock from a hardware store for less than \$3 each. Each sample was about 1 meter long. A wooden or aluminum meter stick would have served equally well. In class, the students hung

standard weights from a wire loop from the center of the beam as shown in Figure 1. The classroom is furnished with wooden tables which are ideal for experiments and demonstrations.



Fig. 1: Weights hang from a beam supported by two classroom tables. Flatstock has a lower tendency to roll than round rods.

II. Procedure

Students used calipers to measure the cross-sectional dimensions of each beam. They placed two worktables 80 cm apart, and bridged the gap with the beam. Next, they measured the height from the floor to the midspan of the beam using a metric tape measure (purchased for \$10 from a hardware store) as shown in Figure 2.

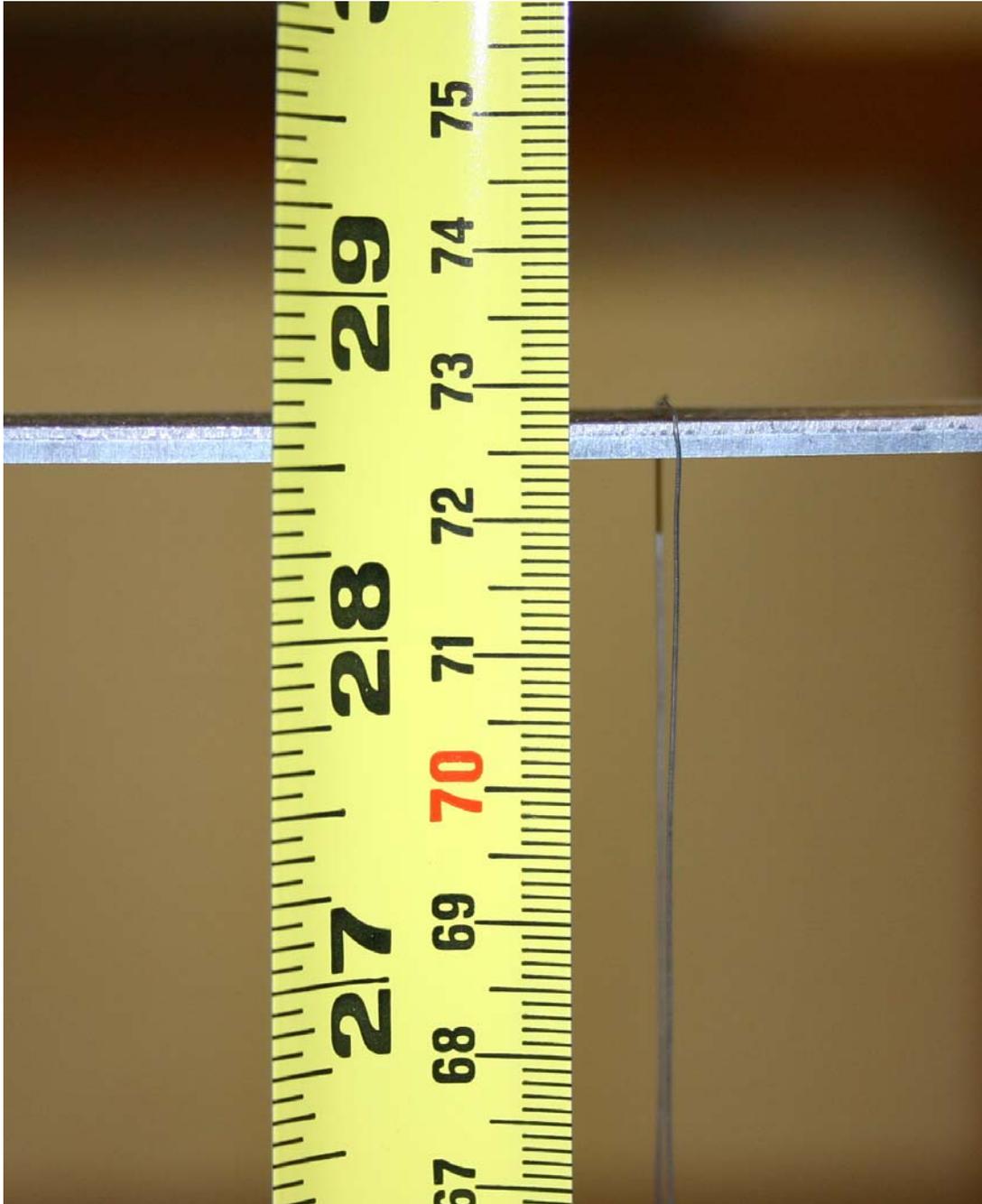


Fig. 2: The height of the bottom of the beam is 724.5 mm off the floor.

They hung a weight from a wire at the center of the span, and again measured the height to the

midspan. Students added more weights, and each time recorded the weight and the height from the floor to the beam. At the end of the test, they removed all the weights and checked the height. This final measurement verified that the beam had not been plastically deformed.

III. Results

Midway through the Strength of Materials course, students learn analytical methods for calculating deflection at the midspan of a simply-supported beam. They also learn how to use prepared formulas from a table (the “formula method”). In our beam experiment, deflection at the midspan is given by [4]:

$$\Delta = W L^3 / 48 E I$$

where Δ = deflection, W = weight, L = arc length of the beam between the support points, E = elastic modulus, and I = second moment of area [5]. This equation ignores the sag due to the weight of the beam. For a rectangular cross-section having a width (base) b and thickness (height) h , the second moment of area (a.k.a. “moment of inertia”) is:

$$I = b h^3 / 12$$

Substituting this equation into the beam deflection equation and solving for E , the elastic modulus is:

$$E = W L^3 / 4 \Delta b h^3$$

The steel flatstock specimen was 25.527 mm wide and 2.819 mm thick; the aluminum flatstock specimen was 25.400 mm wide and 3.175 mm thick. We applied 14 distinct loads to the aluminum strip, and 21 loads to the steel strip. We entered the data into a spreadsheet, which was projected onto a screen in the classroom. We subtracted each height measurement from the initial reading to obtain the deflection, and calculated the modulus for each load, as shown in Table 1. Note that the final zero-load height measurement was identical to the initial zero-load height for both beams, therefore neither beam was plastically deformed in the test.

| Weight (N) | Height from floor (mm) | | Beam deflection (mm) | | Young's modulus (GPa) | |
|------------|------------------------|-------|----------------------|-------|-----------------------|-------|
| | Aluminum | Steel | Aluminum | Steel | Aluminum | Steel |
| 0 | 738 | 738 | 0 | 0 | | |
| 2.5 | 732 | 735 | 6 | 3 | 69 | 197 |
| 3.5 | 729 | 734 | 9 | 4 | 64 | 207 |
| 4.5 | 726 | 732 | 12 | 6 | 62 | 177 |
| 5.5 | 724 | 731 | 14 | 7 | 65 | 186 |
| 6.5 | 721 | 730 | 17 | 8 | 63 | 192 |
| 7.5 | 719 | 728 | 19 | 10 | 65 | 177 |
| 8.5 | 716 | 727 | 22 | 11 | 64 | 183 |
| 9.5 | 713 | 726 | 25 | 12 | 63 | 187 |
| 10.5 | 711 | 724 | 27 | 14 | 64 | 177 |
| 11.5 | 709 | 723 | 29 | 15 | 65 | 181 |
| 12.5 | 706 | 722 | 32 | 16 | 64 | 185 |
| 13.5 | 703 | 721 | 35 | 17 | 64 | 188 |
| 14.5 | 700 | 720 | 38 | 18 | 63 | 190 |
| 15.5 | 697 | 718 | 41 | 20 | 62 | 183 |
| 16.5 | | 717 | | 21 | | 186 |
| 17.5 | | 715 | | 23 | | 180 |
| 18.5 | | 715 | | 23 | | 190 |
| 19.5 | | 713 | | 25 | | 184 |
| 20.5 | | 712 | | 26 | | 186 |
| 21.5 | | 710 | | 28 | | 181 |
| 22.5 | | 710 | | 28 | | 190 |
| 0 | 738 | 738 | 0 | 0 | | |
| Average: | | | | | 64 | 186 |

Table 1: Test data from aluminum and steel flatstock.

Next, we conducted a similar experiment with the round steel rod. For a circular cross-section having a diameter d , the second moment of area is:

$$I = \pi d^4 / 64$$

Substituting this equation into the beam deflection equation and solving for E , the elastic modulus is:

$$E = 4 W L^3 / 3 \pi \Delta d^4$$

The specimen had a diameter of 6.299 mm. We applied 11 distinct loads to the steel rod, as tabulated below. Again, we measured the height with zero load at the end of the test to verify that the rod was not plastically deformed.

| Weight (N) | Height from floor (mm) | Beam deflection (mm) | Young's modulus (GPa) |
|------------|------------------------|----------------------|-----------------------|
| 0 | 738 | 0 | |
| 2.5 | 737 | 1 | 345 |
| 4.5 | 735 | 3 | 207 |
| 6.5 | 733 | 5 | 179 |
| 8.5 | 733 | 5 | 235 |
| 10.5 | 732 | 6 | 242 |
| 12.5 | 730 | 8 | 216 |
| 14.5 | 729 | 9 | 222 |
| 16.5 | 727 | 11 | 207 |
| 18.5 | 727 | 11 | 232 |
| 20.5 | 725 | 13 | 218 |
| 22.5 | 725 | 13 | 239 |
| 0 | 738 | 0 | |
| Average: | | | 231 |

Table 2: Test data from a steel round rod. Excluding the high initial value of 345 GPa, the average modulus was 220 GPa.

IV. Discussion

The equation for modulus of elasticity requires students to pay attention to units: inputs are in N, cm, and mm, while the output is in GPa. I reinforced the importance of using the factor-label method by writing the modulus equation on the chalkboard. In this example, it is aluminum flatstock loaded with 10.5 N.

$$E = \frac{10.5\text{N}(80.0\text{cm})^3}{4(27\text{mm})(25.400\text{mm})(3.175\text{mm})^3} \frac{\text{GPa m}^2}{10^9 \text{ N}} \frac{\text{m}^3}{(100\text{cm})^3} \frac{(1000\text{mm})^5}{\text{m}^5} = 64\text{GPa}$$

Handbook values for the modulus of elasticity range from 68 to 72 GPa for commercially-available aluminum alloys [6], and from 196 to 207 GPa for low-carbon steel [7]. The experimental results are within 12% of published values. These results compare well with previous papers using wire (reference [1] reports a 16% error for copper wire, while reference [2] reports a 3% error for steel wire). These differences provide a discussion point for sources of error. For example, in class I ask the students to calculate the effect of a 1 mm error in measuring beam deflection, a 0.05 mm error in measuring beam thickness, and a 2% error in the weights that hang from the beams. Results are very sensitive to beam dimensions, because the modulus is inversely proportional to the cube of the flatstock thickness and the fourth power of the round rod diameter. For example, if the diameter is mismeasured by 1%, the modulus value will be off by more than 4%.

At the end of the experiment, we discussed ways to improve the test method. One suggestion was to use a meter stick as a test specimen, because the actual arc length could be recorded during the test. Students had some trouble reading the measuring tape because of physical interference between the hanging weights and the tape measure, so we discussed ways to measure beam deflection from above, using a straightedge mounted parallel to the beam.

Students responded very positively to the experiment. They recommended using demonstrations and experiments throughout the semester as a way to reinforce the theory of Strength of Materials.

V. Conclusions

Measuring Young's modulus with a centrally-loaded beam requires less equipment than the popular wire-stretching apparatus, while producing approximately the same error with respect to published values. I spent about \$20 on beams and a measuring tape, which is less than one fifth the cost of a commercially-available strain indicator used in wire stretching tests. While wire tests are limited to materials that can be formed into a wire or monofilament strand (metals, glass, and some polymers), the beam experiment can be performed with wood, metal, polymers, ceramics, and composites. A further refinement would be to use flatstock of various sizes and rod of various diameters to demonstrate that the elastic modulus is independent of the dimensions of the specimen.

This experiment reinforces such skills as data collection and processing, the factor-label method, error analysis, and critique and improvement of test procedures. Students also learn that basic materials properties can sometimes be measured with inexpensive test equipment. After graduation, some of these students may use this technique in the workplace to measure the elastic modulus of a new material, such as a composite, if the test data or sophisticated testing machines are not readily available.

References

- [1] Freeman, W. Larry, and Freda, Ronald F. "A Simple Experiment for Determining the Elastic Constant of a Fine Wire," *The Physics Teacher* **45**, p. 224-227, April 2007.
- [2] Niculescu, Adam, and Shumaker, Russell "Apparatus for Measuring Young's Modulus," *The Physics Teacher* **41**, p. 364-367, September 2003.
- [3] Sargent Welch's model WL0578L sells for \$112, not including the cost of the hooked masses, 2007.
- [4] Roark, Raymond J., and Young, Warren C. *Formulas for Stress and Strain*, 5th ed., McGraw-Hill, p. 97, 1982.
- [5] In the calculations, it is assumed that L = the distance between the tables. For small beam deflections, the arc length is virtually the same as the distance between supports.

[6] *ASM Handbook Vol. 2 – Properties and Selection of Nonferrous Alloys and Special Purpose Materials*, ASM International, p. 62-122, 1990.

[7] Ashby, Michael F., and Jones, David R. H. *Engineering Materials I: An Introduction to their Properties and Applications*, Pergamon Press, p. 31, 1980.

[8] Spiegel, Leonard, and Limbrunner, George *Applied Statics and Strength of Materials*, 4th ed., Pearson Prentice Hall, inside back cover, 2004.